Recurrence quantification analysis of energy market crises: a nonlinear approach to risk management*

Andrii O. Bielinskyi[1,2], Vladimir N. Soloviev[1,3], Viktoria V. Solovieva[2], Serhiy O. Semerikov[1,4,5,6,7] and Michael A. Radin[1]

[1] Kryvyi Rih State Pedagogical University, 54 Gagarin Ave., Kryvyi Rih, 50086, Ukraine
[2] State University of Economics and Technology, 16 Medychna Str., Kryvyi Rih, 50005, Ukraine
[3] Kyiv National Economic University named after Vadym Hetman, 54/1 Peremogy Ave., Kyiv, 03680, Ukraine
[4] Kryvyi Rih National University, 11 Vitalii Matusevych Str., Kryvyi Rih, 50027, Ukraine
[5] Institute for Digitalisation of Education of the National Academy of Educational Sciences of Ukraine, 9 M. Berlynskoho Str., Kyiv, 04060, Ukraine
[6] Zhytomyr Polytechnic State University, 103 Chudnivsyka Str., Zhytomyr, 10005, Ukraine
[7] Academy of Cognitive and Natural Sciences, 54 Gagarin Ave., Kryvyi Rih, 50086, Ukraine
[8] Rochester Institute of Technology, 1 Lomb Memorial Dr, Rochester, NY 14623, United States

Abstract

The energy market is characterized by unstable price dynamics, which challenge the quantitative models of pricing processes and result in abnormal shocks and crashes. We use recurrence quantification analysis (RQA) to analyze and construct indicators of intermittent events in energy indices, where regular patterns are interrupted by chaotic fluctuations, which could signal the onset of crisis events. We apply RQA to daily data of Henry Hub natural gas spot prices, WTI spot prices, and Europe Brent spot prices. Our empirical results show that the recurrence measures capture the distinctive features of crashes and can be used for effective risk management strategies.

Keywords

energy market, recurrence quantification analysis, crash detection, risk management, price dynamics, instability, abnormal shocks

1. Introduction

Crude oil stands as a linchpin in the stability of global economic and financial systems, rendering it a strategic asset for national economic progress [2, 3]. Analyzing the multifaceted determinants impacting crude oil prices becomes paramount for investors, governmental bodies, and stakeholders. These price fluctuations stem from diverse sources, including fundamental

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krivogame@gmail.com (A. O. Bielinskyi); vnsoloviev2016@gmail.com (V. N. Soloviev); vikasolovieva2027@gmail.com (V. V. Solovieva); semerikov@gmail.com (S. O. Semerikov); michael.a.radin@gmail.com (M. A. Radin)

0000-0002-2821-2895 (A. O. Bielinskyi); 0000-0002-4945-202X (V. N. Soloviev); 0000-0002-8090-9569 (V. V. Solovieva); 0000-0003-0789-0272 (S. O. Semerikov); 0000-0001-9951-7955 (M. A. Radin)

factors such as crude oil supply and demand [4], as well as non-fundamental factors like investor sentiment and speculations [5]. Notably, the interplay of the global economic landscape, geopolitical stability among oil-producing nations, and economic policy uncertainty significantly shape crude oil prices.

Given the pivotal role of crude oil in economic advancement, the market's volatile nature has led to substantial economic repercussions, especially for oil-import-dependent countries. Consequently, numerous studies have delved into the drivers of crude oil price volatility, fuelling debates within academia regarding the mechanics of the crude oil market [6, 7, 8]. Amidst this discourse, the intricate risks posed by crude oil price fluctuations, driven by their stochastic and complex nature, have come to the forefront [9, 10, 11, 12].

Two key benchmarks, namely WTI and Brent contracts, typically set oil prices. These benchmarks are favored by hedge funds and traders, resulting in considerable interest in the WTI-Brent pricing structure encompassing futures curve shapes, benchmark price disparities, and market integration levels. Financial institutions actively engage in these markets, amplifying their influence on jet fuel, diesel, heating oil, and gasoline prices. Furthermore, the WTI-Brent spread serves as a foundation for derivative financial products like swaps and options.

While microeconomic theory attributes crude oil prices to supply and demand dynamics, the financialization of oil in the past decade has heightened speculative influences, complicating price determination [13, 14].

The natural gas industry has flourished due to substantial market demand, abundant cost-effective supply, and thriving global trade. Predicting natural gas prices holds critical importance in trading, electric power planning, and regulatory decision-making. Henry Hub (U.S.), NBP (U.K.), and LNG (Japan) now serve as vital international natural gas trading hubs. Among these, Henry Hub boasts the highest liquidity, widest impact, and strongest reflection of supply-demand dynamics. Beyond fundamental factors, natural gas prices are influenced by elements such as extreme weather, geopolitical conflicts, and international relations [15].

Given the nonlinear and non-stationary traits of crude oil and natural gas markets under intricate influences, enhancing the early detection accuracy of market crises remains a pivotal research goal. This paper introduces recurrence analysis-based indicators (indicator-precursors) for this purpose.

2. Methodology of recurrence analysis

In 1890 Poincaré introduced *Poincaré recurrence theorem* [16], which states that certain systems return to their arbitrarily close, or exactly the same initial states after a sufficiently long but finite time. Such property in the case of deterministic behavior of the system allows us to make conclusions regarding its future development.

2.1. Time delay method

The state of the system can be described by the set of variables. Its observational state can be expressed through a d-dimensional vector or matrix, where each of its components refers to a single variable that represents a property of the system. After a while, the variables change, resulting in different system states.

Usually, not all relevant variables can be captured from our observations. Often, only a single variable may be observed. *Thakens' theorem* [17] that was mentioned in previous sections ensures that it‘s possible to reconstruct the topological structure of the trajectory formed by the state vectors, as the data collected for this single variable contains information about the dynamics of the whole system.

For an approximate reconstruction of the original dynamics of the observed system, we project the time series onto a Reconstructed Phase Space [18, 19, 20] with the commonly used time delay method [19] which relied on the *embedding dimension* and *time delay*.

The embedding dimension is being the dimensionality of the reconstructed system (corresponds to the number of relevant variables that may differ from one system to another. The time delay parameter specifies the temporal components of the vector components.

2.2. Recurrence plot

Recurrence plot (RP) have been introduced to study dynamics and recurrence states of complex systems. When we create RP, at first, from recorded time series we reconstruct phase-space trajectory. Then, according to Eckmann et al. [21], we consider a trajectory $\vec{\mathrm{X}}(i)$ on the reconstructed trajectory. The recurrence plot is an array of dots in a $N \times N$ matrix, where dot is placed at (i, j) whenever $\vec{\mathrm{X}}(j)$ is sufficiently close to $\vec{\mathrm{X}}(i)$, and both axes are time axes which mathematically can be expressed as

$$R_{ij} = \Theta(\epsilon - \left\|\vec{\mathrm{X}}(i) - \vec{\mathrm{X}}(j)\right\|), \quad \text{for } i, j = 1, \ldots, N, \tag{1}$$

where $\| \ \|$ is a norm (representing the spatial distance between the states at times i and j); ϵ is a predefined recurrence threshold, and $\Theta(\cdot)$ is the Heaviside function. As a result, the matrix captures a total of N^2 binary similarity values.

Typically, L_p-norm is applied to determine the pairwise similarity between two vectors. According to Webber and Zbilut [22], the L_1-norm (Taxicab metric), the L_2-norm (Euclidean metric), and the L_∞-norm (Chebyshev metric) can serve as candidates for measuring distance between trajectories in phase space.

Also, as it can be seen from equation (1), the similarity between vectors is determined by a threshold ϵ. The choice of $\epsilon > 0$ ensures that all vectors that lie within this radius are similar to each other, and that dissimilarity up to a certain error is permitted [16].

The fixed radius for recurrent states is the commonly used condition, which leads to equally sized ϵ-neighborhoods. The shape in which neighborhoods lie is determined by the distance metric. Applying the fixed threshold with the distance metric, we define recurrence matrices that are symmetric along the middle diagonal. The self-similarity of the multi-dimensional vectors reflects in the middle diagonal, which is commonly referred to as the line of identity (LOI). In contrast, it is not guaranteed that a recurrence matrix is symmetric if the condition of the xed number of nearest neighbors is applied. For specific purposes (e.g., quantification of recurrences), it can be useful to exclude the LOI from the RP, as the trivial recurrence of a state with itself might not be of interest [23].

The main purpose of RP is the visualization of trajectories and hidden patterns of the systems [24, 23].

The dots within RP, representing the time evolution of the trajectories, exhibit characteristic large-scale and small-scale patterns. Large-scale patterns of RP can be classified as

- *homogeneous* – autonomous and stationary systems, which consist of many recurrence points that are homogeneously distributed (relaxation times are short);
- *periodic* – long, uninterrupted, and diagonally oriented structures that represent which indicate periodic behavior. These lines are usually distributed regularly;
- *drift* – systems with patterns paling or darkening from the LOI to the outer corners of RP;
- *disrupted* – systems with drastic changes as well as extreme events in the system dynamics.

The small-scale clusters can represent a combination of *isolated dots* (abrupt events). Similar evolution at different periods in time or in reverse temporal order will present *diagonal lines* (deterministic structures) as well as *vertical/horizontal lines* to inscribe laminar states (intermittency) or systems that paused at singularities. For the quantitative description of the system, such small-scale clusters serve the base of the *recurrence quantification analysis* (RQA).

2.3. Recurrence quantification analysis

The graphic representation of the system suits perfectly for a qualitative description. However, the main disadvantage of graphical representation is that it forces users to subjectively intuit and interpret patterns and structures presented within the recurrence plot. Also, with the increasing size of RP, they can be hardly depicted on graphical display as a whole. As a result, we need to work with separated parts of the original plot. Analysis in such a way may create new defects, which should distort objectivity of the observed patterns and lead to incorrect interpretations. To overcome such limitation and spread an objective assessment among observers, in the early 1990s by Webber and Zbilut [25, 26] were introduced definitions and procedures to quantify RP's complexity, and later, it has been extended by Marwan et al. [27].

The first known measure of the RQA is *recurrence rate*, which measures the probability that the studied process will recur (RR):

$$RR = \frac{1}{N^2} \sum_{i,j=1}^{N} R_{i,j}. \tag{2}$$

Another measure is based on frequency distribution of line structures in the RP. First, we consider the histogram of the length of the diagonal structures in the RP

$$\begin{aligned} P(l) = \sum_{i,j=1}^{N} & (1 - R_{i-1,j-1}) \\ & \times (1 - R_{i+l,j+l}) \prod_{k=0}^{l-1} R_{i+k,j+k}. \end{aligned} \tag{3}$$

The percentage of recurrence points that form diagonal segments of minimal length l_{min} parallel to the main diagonal is the measure of *determinism* (*DET*):

$$DET = \sum_{l=l_{min}}^{N} lP(l) \Big/ \sum_{l=1}^{N} lP(l). \tag{4}$$

Systems that are characterized by long diagonal lines are presented to be periodic. From chaotic signals, we would expect short diagonal lines, and stochastic processes would not present any diagonal lines. Performing the RQA, typically, we rely on the lines with minimal length, which excludes the shorter lines, which may be spurious for characterizing deterministic processes. In our case, $l_{min} = 2$ is considered. In case when $l_{min} = 1$, DET and RR are identical.

Considering diagonal line segments, we can emphasize the longest one – L_{max}. This indicator measures the maximum time that two trajectories remain close to each other and can be interpreted as the maximum prediction time:

$$L_{max} = \max\left(\{l_i \,|\, i = 1, \ldots, N_l\}\right), \tag{5}$$

where $N_l = \sum_{l \geqslant l_{min}} P(l)$ is the total number of diagonal lines.

Divergence (DIV) is the inverse of L_{max} characterizes the exponential divergence of the phase space trajectory [28, 29]:

$$DIV = 1 \Big/ L_{max}. \tag{6}$$

For longer diagonal lines system is more deterministic and, therefore, the measure of divergence is also lower. The smaller L_{max}, the more divergent are trajectories and more chaotic the studied system. According to Eckmann et al. [21], *DIV* can be used to estimate the largest positive Lyapunov exponent.

Another measure which is related to the diagonal line segments is the *average diagonal line length* (L_{mean}):

$$L_{mean} = \sum_{l=l_{min}}^{N} lP(l) \Big/ \sum_{l=l_{min}}^{N} P(l) \tag{7}$$

It can be interpreted as the mean prediction horizon of the system, and it measures average time that two trajectories remain close to each other.

Using the classic Shannon entropy, we can measure the hidden complexity of recurrence structures in the RP. In accordance with this study, the entropy of diagonal line histogram (*DLEn*) is of the greatest interest. It can be defined as:

$$DLEn = -\sum_{l=l_{min}}^{N} p(l) \ln p(l) \tag{8}$$

and

$$p(l) = P(l) \Big/ \sum_{l=l_{min}}^{N} P(l), \tag{9}$$

where $p(l)$ captures the probability that a diagonal line has exactly length l, and *DLEn* reflects the complexity of deterministic structure in the system. The more uniform is the frequency distribution of diagonal lines, the higher the value of *DLEn*. If there is predominant deterministic behavior with a particular period l, then *DLEn* becomes lower.

As it was mentioned, the RP structure consists of vertical (horizontal lines). For them Marwan and Webber [30] proposed additional recurrence measures. The first of them is the *laminarity* (*LAM*) Analogously to the equation (4), which measures the percentage of diagonal lines with minimal length l_{min} in the RP, we can calculate the fraction of recurrence points forming vertical structures of minimal length v_{min}:

$$LAM = \sum_{v=v_{min}}^{N} vP(v) \Big/ \sum_{v=1}^{N} vP(v) \tag{10}$$

with

$$\begin{aligned} P(v) = \sum_{i,j=1}^{N} & \left(1 - R_{i,j-1}\right) \\ & \times \left(1 - R_{i,j+v}\right) \prod_{k=0}^{v-1} R_{i,j+k} \end{aligned} \tag{11}$$

as the histogram of lengths of vertical lines.

Since it measures the overall amount of vertical lines, it characterizes the percentage of laminar states within the system. If *LAM* increases, then there are more vertical or diagonal structures than isolated recurrent points.

Similarly to L_{max}, we can define the measure which will indicate the maximum time that a system holds an unchangeable pattern – the *maximal vertical lines length* (V_{max}):

$$V_{max} = \max\left(\{v_i \,|\, i = 1, \ldots, N_v\}\right), \tag{12}$$

where $N_v = \sum_{v \geqslant v_{min}} P(v)$ is the total number of vertical lines.

Vertical line divergence (*VDIV*) is the analogous to (6), which can be related to the rate of divergence from laminar state:

$$VDIV = 1 \Big/ V_{max}. \tag{13}$$

Consequently, we can define the average time that two trajectories remain at a specific state – *trapping time* (*TT*):

$$TT = \sum\nolimits_{v=v_{min}}^{N} vP(v) \Big/ \sum\nolimits_{v=v_{min}}^{N} P(v). \tag{14}$$

For high *TT* values we would expect the system to consist of more laminar states, whereas low *TT* values would indicate abrupt changes in the system's dynamics.

The variability of laminar states with different duration time can be measured in the same way as for diagonal lines – using Shannon entropy. The complexity of vertical lines can be measures according to the following equation:

$$VLEn = -\sum_{v=v_{min}}^{N} p(v) \ln p(v) \tag{15}$$

with

$$p(v) = P(v) \Big/ \sum_{v=v_{min}}^{N} P(v) \tag{16}$$

indicating the probability of a vertical line to have length $v \geqslant v_{min}$.

In the same manner, we can quantify the variation (complexity) of abrupt changes during the studied periods in the energy markets. Regarding equation (7), we can quantify the average time of divergence when two trajectories in the phase-space remain out of recurrence threshold ϵ. This measure can be called as *average white vertical line length* (WVL_{mean}):

$$WVL_{mean} = \sum_{w=w_{min}}^{N} wP(w) \Big/ \sum_{w=w_{min}}^{N} P(w), \tag{17}$$

where $P(w)$ is the frequency of white vertical lines in the RP. This measure can be interpreted as the mean horizon of unpredictability of the system.

This kind of complexity is associated with the white vertical lines in the RP and can be quantified in the following way:

$$WVLEn = -\sum_{w=w_{min}}^{N} p(w) \ln p(w) \tag{18}$$

with

$$p(w) = P(w) \Big/ \sum_{w=w_{min}}^{N} P(w) \tag{19}$$

indicating the probability of a white vertical line to have length $w \geqslant w_{min}$.

The further measure is based on the ration between *DET* and *RR*, and known as *ratio* (DET/RR):

$$DET/RR = N^2 \sum_{l=l_{min}} P(l) \Big/ \left(\sum_{l=1}^{N} lP(l) \right)^2 \tag{20}$$

In the same manner, we can define another measure which is based on the ratio between *LAM* and *DET*:

$$\begin{aligned} LAM/DET = & \sum_{v=v_{min}}^{N} vP(v) \cdot \sum_{l=1}^{N} lP(l) \\ & \Big/ \sum_{v=1}^{N} vP(v) \cdot \sum_{l=l_{min}}^{N} lP(l). \end{aligned} \tag{21}$$

This measures can be used to uncover hidden transitions in the dynamics of the system [25].

3. Results and analysis

Regarding previous studies, we present additional analysis on co-movement between 3 energy-related indices and construct indicators or indicators-precursors based on the using recurrence analysis.

The presented work uses daily data of Henry Hub natural gas spot prices (US$/MMBTU) ranged from 7 February 1997 to 18 October 2022; Cushing, OK WTI spot prices FOB (US$/BBL) ranged from 20 May 1987 to 17 October 2022; Europe Brent spot prices FOB (US$/BBL) ranged from 20 May 1987 to 17 October 2022 [31, 32].

In figure 1 are presented:

- the dynamics of the initial time series;
- standardized returns, where returns can be calculated as $G(t) = [x(t + \Delta t) - x(t)]/x(t)$ and their standardized version as $g(t) = [G(t) - \langle G \rangle]/\sigma$;
- probability density function of the standardized returns.

We can see that most periods in energy markets are defined by events that exceed $\pm 3\sigma$. Both WTI and Brent returns are characterized by much more extensive crashes. Previous studies pointed out that such events are located in fat-tails of the probability distribution. Such crashes are the main source of high complexity and non-linearity in the studied systems.

Most of our results are based on the sliding window approach. The idea here is to take a sub-window of a predefined length w. For that sub-window, we perform recurrence quantification analysis, get necessary indicators that are appended to the array. Then, the window is shifted by a predefined time step h, and the procedure is repeated until the time series is completely exhausted.

We have performed RQA under sliding window procedure for standardized returns and standardized initial time series [33, 34, 35, 36, 37, 38]. We have found that standardized initial time series better expresses internal complexity and recurrent properties of the energy market indices.

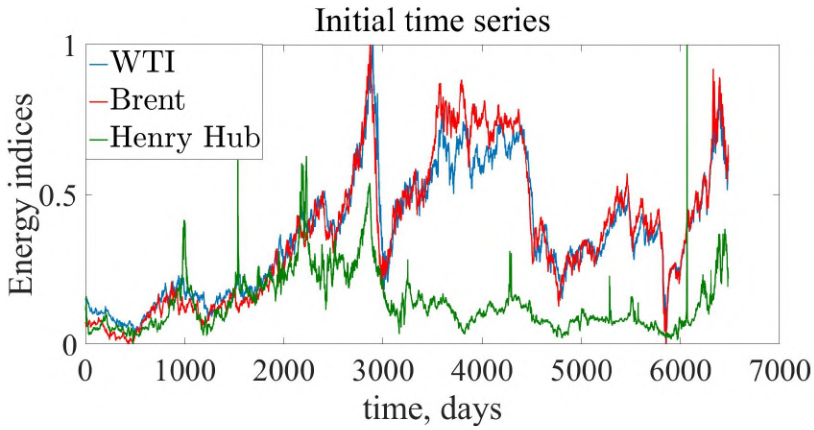
RQA was performed for the following parameters:

- embedding dimension $d_E = 1$;
- time delay $\tau = 1$;
- recurrence threshold $\epsilon = 0.3$;
- L_2-norm as a candidate for measuring distance between trajectories in phase space;
- minimum diagonal line length $l_{min=2}$;
- minimum vertical line length $v_{min} = 2$;
- minimum white vertical line length $w_{min} = 2$;
- sliding window length $w = 500$ days;
- sliding window time step $h = 1$ day.

Worth to mention that the experiments were performed for sliding window lengths of 250 days and 500 days. We have chosen the second option since it represents a more reliable and smoother dynamics of all the presented indicators. All described measures result into highly volatile variation with the sliding window of 250 days that difficult to interpret.

In figure 2 are presented RPs for the studied series.

Recurrence plots in figure 2 represent that the studied energy markets are highly inhomogeneous. As it was expected, nonlinear structure of WTI and Brent is presented to be very similar, comparing to Henry Hub. Recurrence structure of all indices varies across time. They do not



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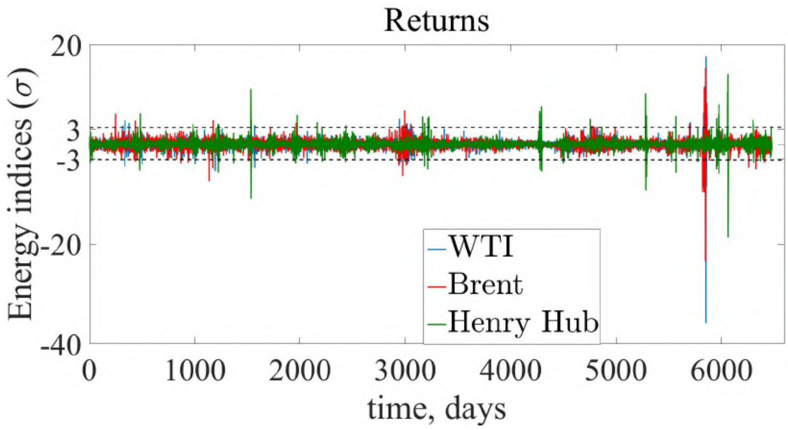

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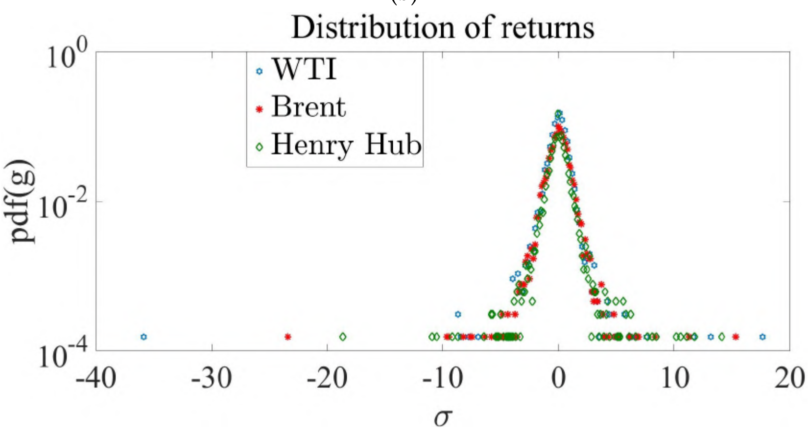

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Figure 1: Initial time series (a), standardized returns (b), and pdf of standardized returns of WTI spot prices (WTI), Europe Brent spot prices (Brent), and Henry Hub natural gas spot prices (Henry Hub).

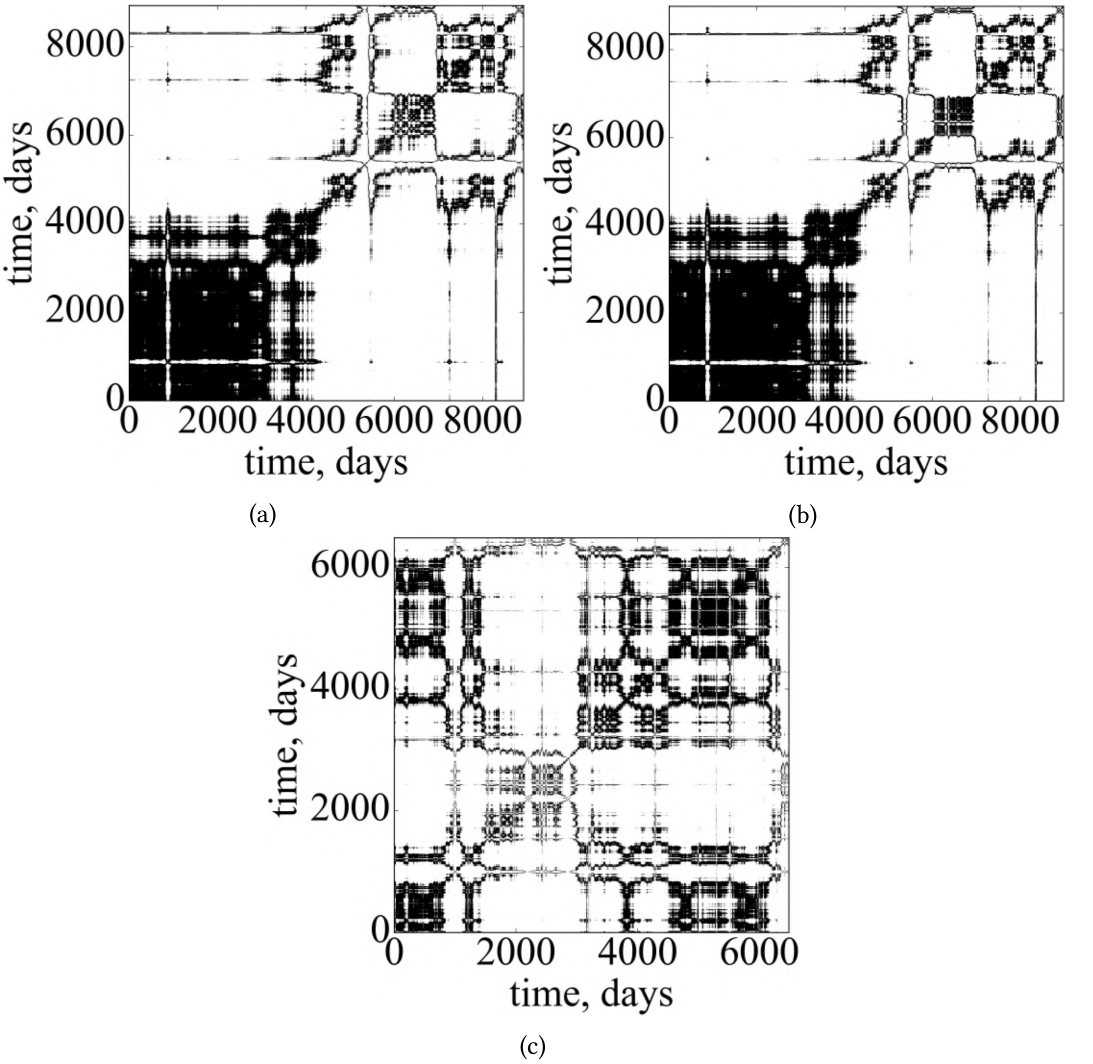
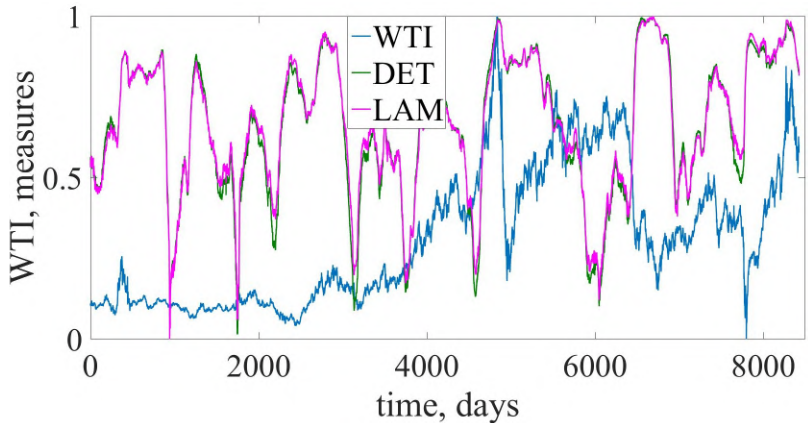


Figure 2: Recurrence plots calculated for WTI (a), Brent (b), and Henry Hub (c) standardized time series.

follow a certain pattern, presented to be non-periodic, and there are differences in the patterns that concern the frequency of their appearance, shape, and size. It should be noticed that for the oil markets first 4000 days are presented to be highly recurrent, while the remaining days seem to be more volatile, which is indicated by high proportion of white regions. The recurrence structure of Henry Hub index is presented to be more uniformly distributed. The variations of recurrence patterns should be more noticeable during crashes. Recurrence quantitative indicators should give a more accurate representation of the complex, chaotic structure of the studied markets.

Figure 3 represents recurrence measures of determinism (*DET*) and laminarity (*LAM*).

In figure 3 we see that *DET* and *LAM* increase during crisis events of all markets. We may conclude that those critical states are characterized by high degree of laminarity and



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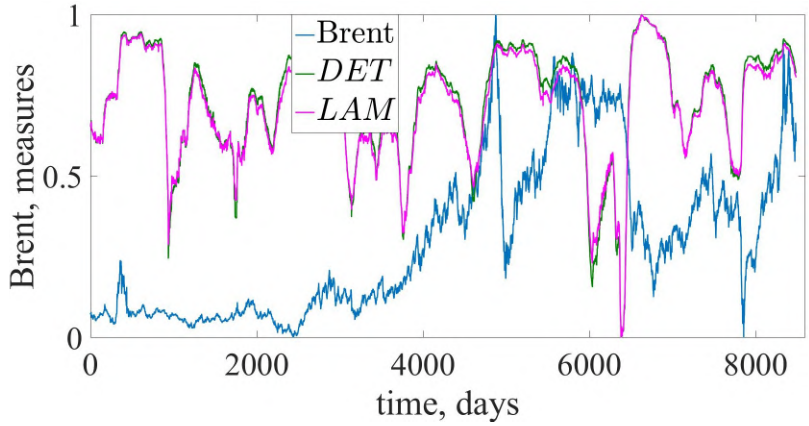

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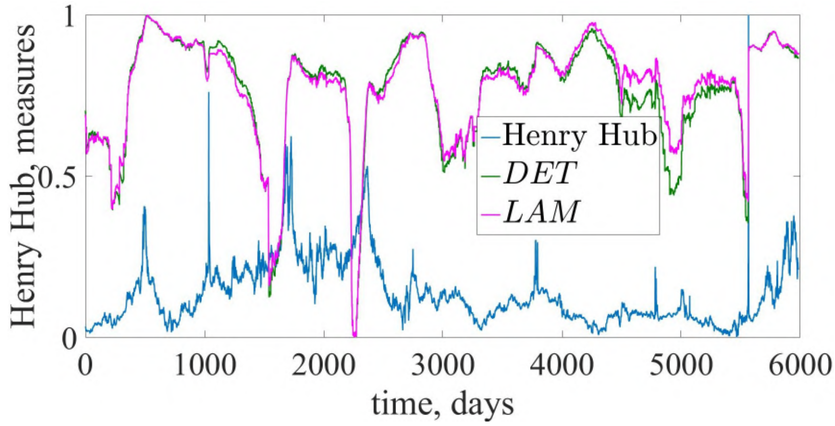

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Figure 3: Recurrence measures of determinism (*DET*) and laminarity (*LAM*) calculated for WTI (a), Europe Brent (b), and Henry Hub (c) indices.

determinism. Crashes are presented to be highly complex and deterministic. Their degree of predictability becomes higher, and corresponding recurrence measures seem to be indicators or even indicators-precursors of such changes.

Figure 4 represents recurrence measures of ratios DET/RR and LAM/DET.

From figure 4 we can see that both measures decrease during crisis events of energy indices. For ratio DET/RR we may say that the overall percentage of recurrence points in RP becomes higher than the percentage of only diagonal structures in RP. For ratio LAM/DET we see precisely the same behavior during crashes, i.e., it starts to decline during crisis or even in advance. Thus, it can be seen that the overall determinism of the system during crashes is much higher than the degree of laminarity.

Figure 5 shows recurrence measures of diagonal (DIV) and vertical line ($VDIV$) divergences.

Figure 5 demonstrates that the divergence of deterministic and laminar structure of energy-related markets becomes lower during critical states. Since both measures are inverse quantities to maximum diagonal and vertical line length (L_{max} and V_{max}), such behavior has to be obvious. Previous measures have made it clear to us that the crisis phenomena of energy indices are characterized by a high degree of determinism and laminarity. In this case, the lengths of diagonal and vertical lines should also increase, which indicate an increase in the horizon of predictability and immutability.

Figure 6 represents recurrence measures of recurrence rate (RR), average diagonal line length (L_{mean}), and trapping time (TT).

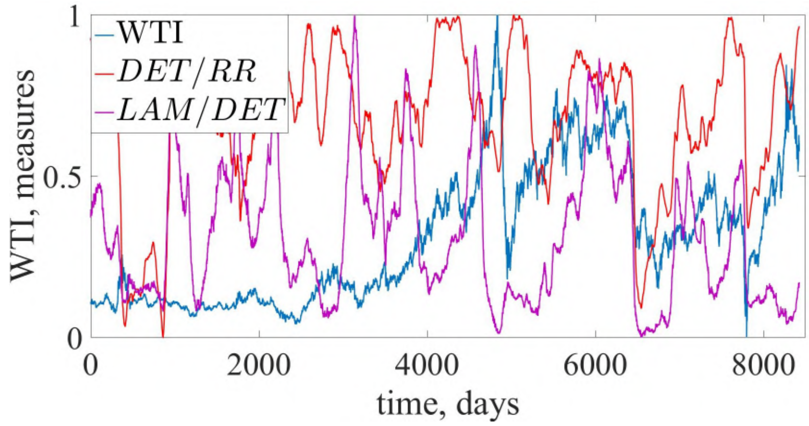
In figure 6 we see that recurrence rate increases during crisis phenomena. This means that the total number of trajectories in the phase space that are close enough to each other becomes larger on the eve of a crisis or at the moment of its onset. Thus, the probability of recurrence state increases during crash. Regarding previous measures, RR and L_{mean}, we see that the average degree of predictability during crisis increases. The same can be seen for trapping time: average degree of changeability increases during crashes. Based on this indicator, we may conclude that the system is ‘trapped’ in a state of crisis.

Figure 7 presents recurrence measures of average white vertical line length (WVL_{mean}), and diagonal, vertical and white vertical line entropies ($DLEn$, $VLEn$, and $WVLEn$).

From figure 7 we can see that all the presented quantitative measures of recurrence begin to increase during crises, indicating a special state of the market at these points in time. The average white vertical line length shows that crisis events are characterized not only by the determinism of the dynamics of market movement, but also by the dissimilarity of these events to many previous ones, since the length of the white vertical lines is becoming an increasing trend. It can also be said that the market represents a much more deterministic structure than a laminar one. Also, the degree of volatility of these events can knock the market dynamics out of the limits of the epsilon value.

The diagonal line entropy also shows an increasing trend. Since the Shannon entropy is maximal with a uniform distribution, it can be concluded that the collapse events of energy indices are characterized by different horizons of predictability. That is, in the pre-crisis dynamics there is no black diagonal line of the same length, which is the dominant one. During a crisis, horizons of determinism appear, which gain even more weight if compared with the rest.

The vertical line entropy increases similarly to $DLEn$. We may assume that similarly to



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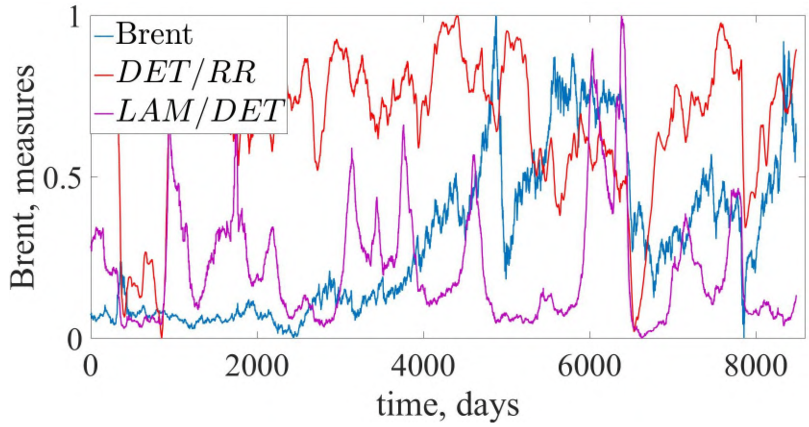

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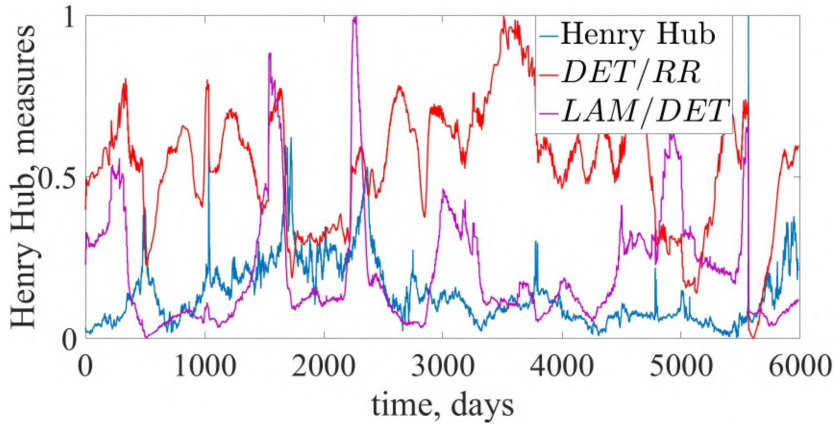

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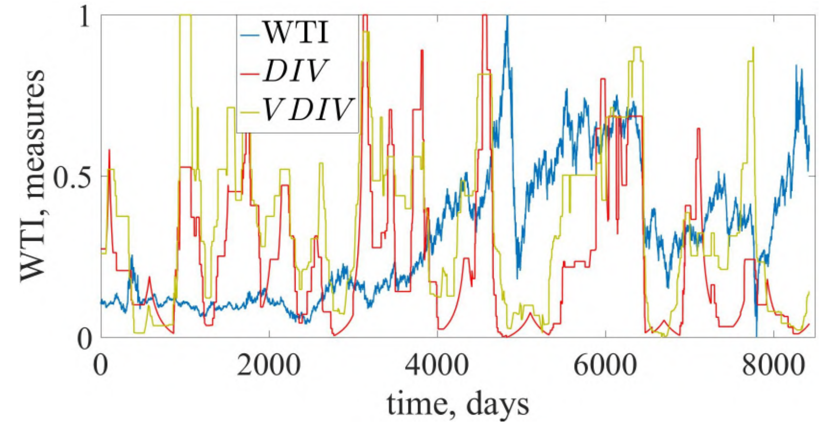
Figure 4: Recurrence measures (DET/RR) and (LAM/DET) calculated for WTI (a), Europe Brent (b), and Henry Hub (c) indices.



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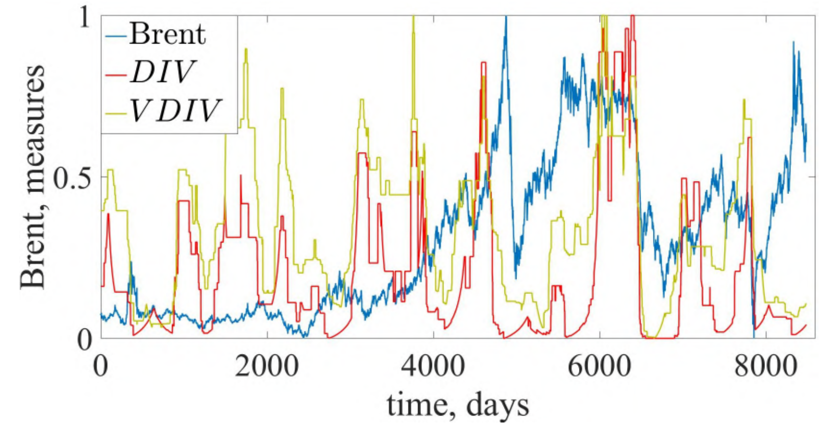

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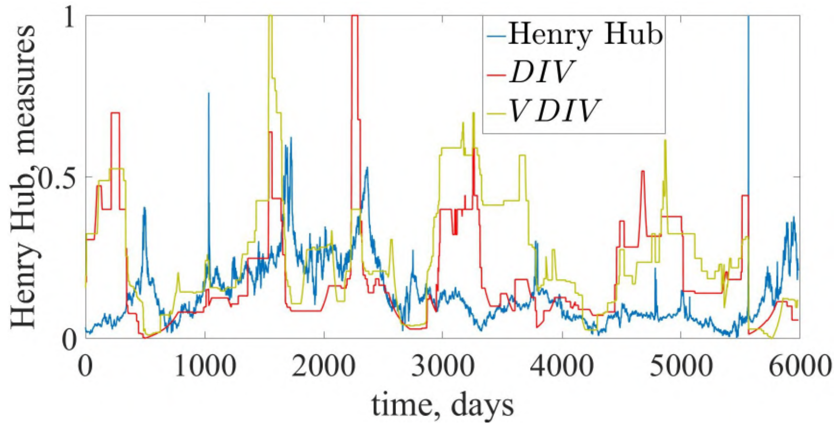

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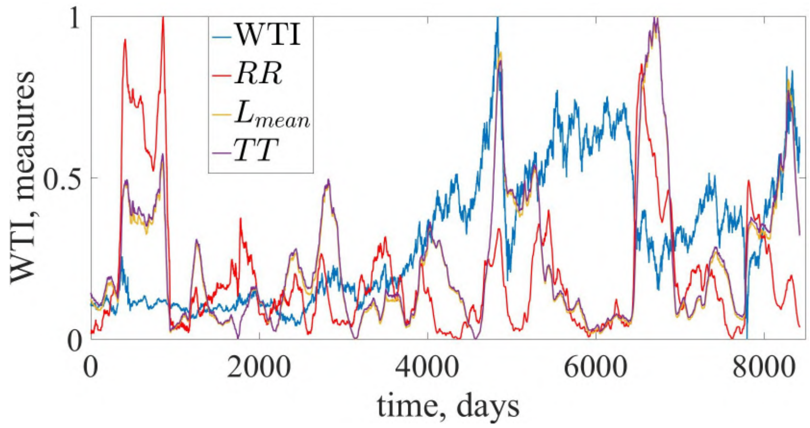
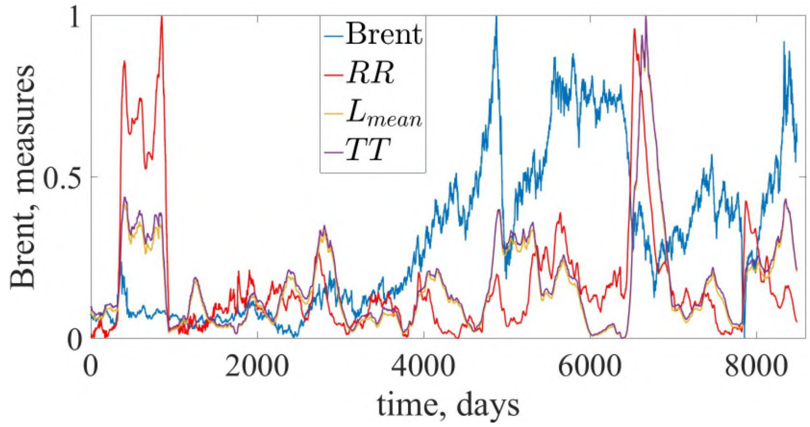
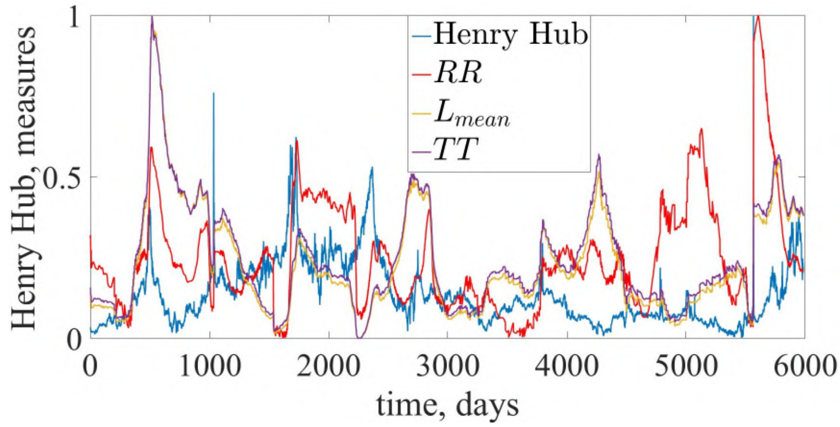
Figure 5: Recurrence measures of diagonal line divergence (DIV) and vertical line divergence ($VDIV$) calculated for WTI (a), Europe Brent (b), and Henry Hub (c) indices.



(a)

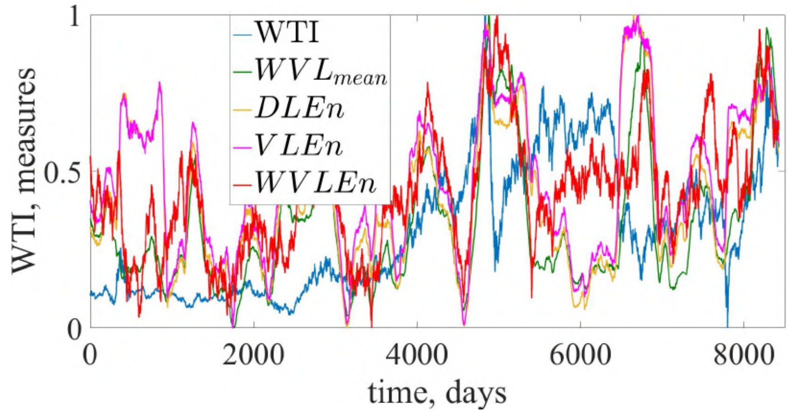


(b)

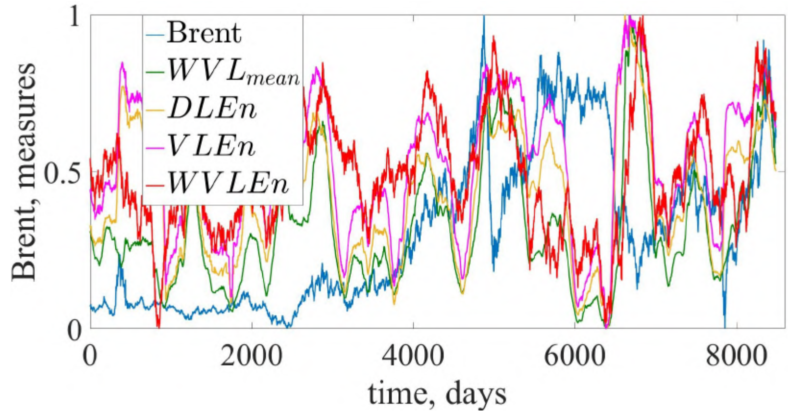


(c)

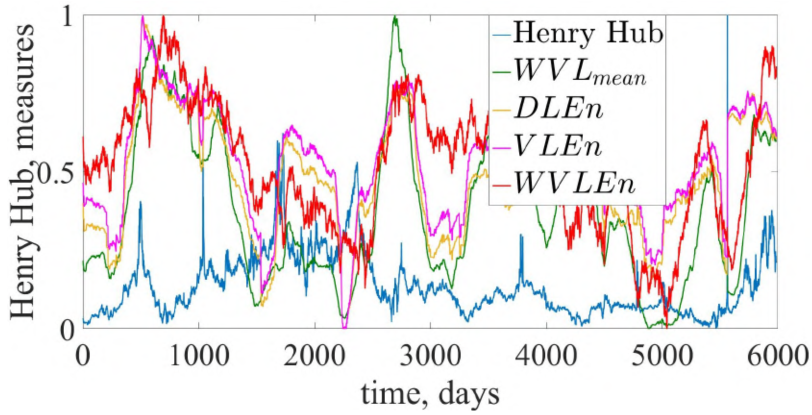
Figure 6: Recurrence measures of recurrence rate (RR), average diagonal line length (L_{mean}), and trapping time (TT) calculated for WTI (a), Europe Brent (b), and Henry Hub (c) indices.



(a)



(b)



(c)

Figure 7: Recurrence measures of average white vertical line length (WVL_{mean}), diagonal line entropy ($DLEn$), vertical line entropy ($VLEn$), and white vertical line entropy ($WVLEn$) calculated for WTI (a), Europe Brent (b), and Henry Hub (c) indices.

diagonal lines laminar states have different horizons of invariability during crash events, and these horizons of invariability have greater tendency to uniform distribution.

The white vertical line entropy increases similarly to other entropies. This dynamics is consistent with the WVL_{mean} measure.

4. Conclusions

This study has delved into the intricate, nonlinear, and nonstationary dynamics of the oil and gas markets through the lens of recurrence analysis. Leveraging daily data spanning from 7 February 1997 to 18 October 2022 for Henry Hub natural gas spot prices, from 20 May 1987 to 17 October 2022 for WTI spot prices, and corresponding data for Europe Brent spot prices, we draw several significant conclusions from our empirical findings.

Firstly, our analysis of recurrence plots reveals the inherent inhomogeneity within the studied markets. Notably, the nonlinear structures of WTI and Brent exhibit remarkable similarities when contrasted with Henry Hub. Furthermore, recurrence patterns for all indices exhibit temporal variations, demonstrating differences in the frequency, shape, and size of black- and white-dot patterns across time.

From quantitative measures of complexity, the following insights emerge:

1. **Characteristics of Crash Events**: Crashes in energy-related indices exhibit a pronounced degree of both laminarity and determinism, indicating a high level of complexity and determinism during these events.
2. **Determinism vs. Laminarity**: The percentage of recurrence points surpasses that of only diagonal structures during crises. While the overall degree of determinism outweighs laminarity, a higher percentage of diagonal lines during crises highlights their significance.
3. **Divergence During Critical States**: The divergence between deterministic and laminar structures diminishes during critical states, indicating increased repeatability in the dynamics of the studied systems. This suggests that phase-space trajectories converge during financial crises.
4. **Recurrence Measures during Crises**: Measures like recurrence rate, mean diagonal line length, and trapping time increase during crisis periods. This implies a larger number of closely situated trajectories in phase space, raising the probability of recurrence states and predictability. Greater presence of vertical lines signifies the system being 'trapped' in a crisis state.
5. **Entropy-based Measures**: Entropy-based measures, especially white vertical line measures, reveal intricate nonlinear patterns in energy-related indices that encompass determinism, laminarity, and dissimilarities reflected in white lines.

The approach applied to WTI, Brent, and Henry Hub indices underscores the energy market's nature as an open, chaotic, nonlinear system intricately intertwined with diverse technical and fundamental factors. While recurrence plots and recurrence quantification analysis offer promising outcomes for crisis prediction and early-warning indicator construction, their practical application in trading strategies and autonomous trading bots necessitates further refinement.

Furthermore, for accurate crisis forecasting, the integration of proposed indicators (indicators-precursors) with specific forecasting models is imperative [11, 39, 15, 12, 40, 41, 42]. This convergence seems particularly promising at the intersection of artificial intelligence and fuzzy logic methods [43, 44, 45, 46, 47, 48, 49].

Simultaneously, we aim to explore cross-recurrences between energy indices and diverse technical and fundamental indicators utilizing cross- and joint-recurrence quantification analysis [50, 51, 52]. This avenue holds potential for further unraveling the intricate relationships within energy markets.

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