UDC 622.794.7:621.3.0

https://doi.org/10.33271/nvngu/2021-3/051

O. V. Zamytskyi,
orcid.org/0000-0002-8113-6369,
N. O. Holiver,
orcid.org/0000-0002-9252-2839,
N. V. Bondar,
orcid.org/0000-0002-8713-265X,
S. O. Kradozhon,
orcid.org/0000-0001-8286-1389

Kryvyi Rih National University, Kryvyi Rih, Ukraine, e-mail: kradozhon.so@knu.edu.ua

MATHEMATICAL MODEL OF THE PROCESS OF DRYING FINE DISPERSED MATERIALS UNDER THE INFLUENCE OF ALTERNATING ELECTRIC CURRENT

Purpose. Establishing the dependences and determining rational parameters of the process of drying fine materials by direct influence of an electric current.

Methodology. In the work, theoretical, analytical, empirical, and experimental methods as well as methods of mathematical statistics are used. Mathematical modeling of the process occurring during drying of finely dispersed materials by direct influence of alternating current is carried out.

Findings. As a result of the research on the basis of physical representations of the process of drying capillary-porous material, a mathematical model is designed connecting temperature and moisture content in a plate from capillary-porous material by means of equations of mathematical physics.

Originality. For the first time dependence has been obtained on the temperature and moisture content of the time and spatial coordinates of drying by passing an electric current through the layer of moist capillary-porous material, a feature which is both simultaneous accounting of thermal and diffusion processes in the material that can increase the accuracy of calculations and establish rational parameters of drying.

Practical value. The obtained dependences are used when developing calculation methods and designing an industrial drying plant.

Keywords: *mathematical model, fine materials, drying*

Introduction. The constant deterioration of iron ore necessitates the processing of hard-to-enrich minerals, requiring deep enrichment, and complicates drying. The use of thermal drying for moisture adjustment is associated with a sharp increase in investment and consumption of scarce energy, which in today's raw materials market and economic situation is impractical. More than 85 % of materials in all industries are subject to drying in the dispersed and dispersing state. The search for new methods for drying fine materials is an important stage in the development of industry. Drying is the most energy-intensive of all technological processes, so special attention should be paid to the energy performance of the device when choosing the type of a dryer. The development of optimal technical solutions and technological modes of drying should be associated with maximum process efficiency. Electric drying of dispersed materials reduces the energy consumption of the process and increases its productivity.

Literature review. Currently, the issue of increasing the productivity of production processes and the search for new methods of optimization in the mining industry is of great importance [1].

Excessive moisture of iron ore prevents its transportation and further use. When the moisture level of the material exceeds a certain limit, it causes delays and large financial losses in production [2]. Also, pre-drying of the sintering mixture has a positive effect on the productivity and quality of iron ore sintering [3].

Let us consider in more detail the methods of drying using electric current. Let us start with dielectric drying with high frequency currents.

Drying with high frequency currents is based on heating dielectrics and semiconductors in a rapidly changing electric field. Such a field, acting on the dielectric, causes the rotational and oscillating motion of its molecules. The resulting molecular friction is converted into heat, the amount of which is proportional to the frequency of the current.

Of all the power supplied to the capacitor, part – the active power – is absorbed by the material, turning into heat. The remaining part of the power does not generate thermal energy and is marked as reactive. The ratio of active power to reactive characterizes the energy loss and is expressed by a coefficient called the tangent of the loss angle. The amount of total energy used largely depends on the dielectric constant of the material. Therefore, all the loss of electrical energy when heating dielectrics is usually expressed as a loss factor. Thus, the main characteristics of the material that determine its heating are the tangent of the loss angle and the dielectric constant of the material. The rapid formation of large amounts of heat at the locations of moisture in the material is a hallmark of heating in a high-frequency electric field. This is especially true of wet materials, as water has a very high dielectric constant.

As the moisture content of the material decreases, the loss factor decreases. In practice, this means that when drying flat materials, wetter areas will preferably be heated until the moisture content is equalized throughout the volume. When dried in an electric field of high frequency, the material is heated from the inside. As a result, the temperature gradient, pressure gradient and moisture are directed from the inner layers of the material to its surface. The mechanism of the drying process depends on the intensity of heating.

The disadvantage of this method is the high energy consumption of dehydration and low performance [4].

There is a method for drying products made of capillary-porous materials by placing them in a layer of dispersed electrically conductive compressed material and heating them by passing through the latter an electric current using the accumulated heat [5]. This method is combined and its disadvantages are high energy consumption, as energy is spent on heating the material and on the production of coolant, and low productivity, as the process is periodic.

Microwave drying is in principle close to drying with high frequency currents, but is carried out at higher frequencies (from 460 to 915–2500 MHz).

Equipment for microwave drying of bulk materials can be used in food, chemical and other industries. The microwave unit for drying bulk materials comprises a frame, a drying chamber, a microwave generator, a radiator in the form of two meandering lines mounted for rotation around an axis, a lid, an adjuster, a drive, a cylindrical screen with holes and an air supply unit connected with a cavity between the frame and the screen.

The technical result is to ensure maximum scattering of microwave energy by the volume of the drying chamber, the uniformity of moisture removal from the processed material, increasing the drying efficiency.

The disadvantage of this method is the high cost of magnetron generators and the complexity of controlling the drying process (in particular, the temperature of the material) [6, 7].

Finally, let us consider another method for drying dispersed materials using electric current. In contrast to the method using a microwave, where electrical energy is converted into wave energy; in this method, the electric current is passed directly through a layer of wet material [8]. The wet material is brought into contact with the electrodes and connected directly to the electrical circuit through which an electric current is passed. When electric current passes through a wet material, the latter releases thermal energy which leads to heating and evaporation of moisture. Due to the fact that when the humidity of the material to be dried decreases, the amount of current flowing through the material and the amount of heat released spontaneously decreases, control over the drying process is carried out by the amount of current flowing in the circuit. Vacuum treatment or purging of the material with compressed air or other gas simultaneously with the passage of electric current facilitates the removal of moisture vapor and accelerates the drying process. The method can be used in various fields of technology, mainly in the chemical and metallurgical industry, for drying such bulk materials which in the wet state are conductors of electric current, i.e. contain electrolyte moisture [9]. The advantage is the prospect of developing a method for determining intermediate values of moisture content in the dried material. Disadvantages include the need to ensure a high level of electrical safety; significant dependence of the rate of dehydration and the final moisture content of the material to be dried on the electromechanical properties of the material and the amount and composition of impurities in the moisture.

This method uses the lowest consumption of conventional fuel to evaporate 1 kg of moisture. Another advantage is that the process takes a short time and has low economic costs when implemented in production. And low emissions of polluted gases into the atmosphere help to improve the environmental situation. Thus, determining the dependences of the drying process by internal heat sources, which can significantly increase the energy efficiency of the drying process of fine materials, is an urgent scientific task.

Purpose. In the process of drying fine dispersed materials by the direct influence of alternating electric current, a number of heat and mass transfer processes take place. Under the influence of electric current, the process is characterized by the spread of a temperature gradient from the inside to the surface [10].

Let us consider heating a plate (Fig. 1) with a thickness of 2δ of wet capillary-porous material with moisture content at temperature and basic physical parameters λ, *c*, ρ. At the initial time, it is placed in a medium with a temperature.

Heat exchange with the environment occurs according to Newton-Richman's law (boundary condition of the third kind). Inside the plate there is a heat source, the specific power of which is equal to *Qv*.

At small thickness of a plate in comparison with length and width it is considered to call such plate unlimited. You need to find the distribution of temperature and moisture content over the thickness of the plate.

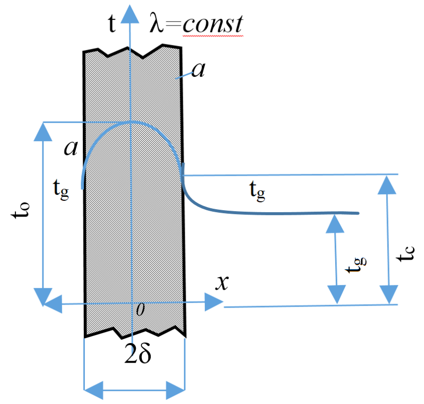

Fig. 1. Thermal conductivity of a flat plate in the presence of internal heat sources

The analytical description of the thermal conductivity process will include the differential equation and the conditions of unambiguity.

Mathematically, the problem is formulated below (the origin of coordinates is in the middle of the plate)

$$\frac{\partial t}{\partial \tau} = a\frac{\partial^2 t}{\partial x^2} + \frac{r'\varepsilon}{c_v}\frac{\partial u}{\partial \tau} + \frac{Q_v}{c_v\rho_0}; \qquad (1)$$

$$\frac{\partial u}{\partial \tau} = a_{m2}\frac{\partial^2 u}{\partial x^2} + a_{m2}\delta_2\frac{\partial^2 t}{\partial x^2} + \varepsilon\frac{\partial u}{\partial \tau}; \qquad (2)$$

$$\frac{\partial P}{\partial \tau} = a_P\frac{\partial^2 P}{\partial x^2} + \varepsilon \cdot P_n\frac{\partial u}{\partial \tau}; \quad (\tau > 0; -\delta < x < \delta); \qquad (3)$$

$$t|_{\tau=0} = t_0; \quad u|_{\tau=0} = u_0; \qquad (4)$$

$$\left.\frac{\partial t}{\partial x}\right|_{x=0} = 0; \quad \left.\frac{\partial u}{\partial x}\right|_{x=0} = 0; \qquad (5)$$

$$-\lambda\left.\frac{\partial t}{\partial x}\right|_{x=\delta} + \alpha(t_c - t|_{x=\delta}) = \rho\cdot q_2'; \qquad (6)$$

$$a_{m2}\rho_0\left.\frac{\partial u}{\partial x}\right|_{x=\delta} + a_{m2}\rho_0\delta_2\left.\frac{\partial t}{\partial x}\right|_{x=\delta} = -q_2', \qquad (7)$$

where *t* is plate temperature, °C; τ – time, s; α is the coefficient of thermal conductivity, m²/s; ε is the liquid-vapor phase transformation coefficient; c_v is specific isochoric heat capacity, j/kg·K; Q_v is specific power dissipated by internal heat sources, W/m³; *r'* is the specific heat of vaporization, j/kg; ρ_0 is density of dry material, kg/m³; a_{m2} is the diffusion coefficient of the liquid, m²/s; δ_2 is the relative coefficient of thermal diffusion, kg/kg · K; *u* is moisture content, kg/kg; u_n is moisture content of the medium, kg/kg; a_p is the convective diffusion coefficient of steam, m²/s; P_n is vapor pressure of the material at a given moisture content, *PA*; ρ is the radius of the capillary meniscus, m; q'_2 is mass transfer intensity, kg/m² · s; λ is the coefficient of thermal conductivity, w/m² · K; α is the heat transfer coefficient, w/m² · K; t_c is ambient temperature, °C; t_0 is the initial temperature of the plate, °C; u_0 is initial moisture content of the plate, kg/kg.

According to the problem to be solved, we assume that the evaporation of moisture occurs only on the surface of the plate. The criterion of phase shift ε = 0 and the transfer of matter occurs only due to the transfer of liquid.

Then the system of differential equations (1–3) will take the form

$$\frac{\partial t}{\partial \tau} = a\frac{\partial^2 t}{\partial x^2} + \frac{Q_v}{c_v\rho_0}; \qquad (8)$$

$$\frac{\partial u}{\partial \tau} = a_{m2}\frac{\partial^2 u}{\partial x^2} + a_{m2}\delta_2\frac{\partial^2 t}{\partial x^2}; \qquad (9)$$

$$\frac{\partial P}{\partial \tau}=a_P\frac{\partial^2 P}{\partial x^2}. \quad (10)$$

The analysis of the solved problem shows that the ultimate goal is to find the set humidity in the investigated plate from wet capillary-porous material. In this case, the partial derivatives of time are equal to zero and the system of differential equations (8, 9) is written as a system of ordinary differential equations

$$a\frac{d^2t}{dx^2}+\frac{Q_v}{c_v\rho_0}=0; \quad (11)$$

$$\frac{d^2u}{dx^2}+\delta_2\frac{d^2t}{dx^2}=0. \quad (12)$$

The boundary conditions take the form

$$\left.\frac{dt}{dx}\right|_{x=0}=0\cdot\left.\frac{du}{dx}\right|_{x=0}=0; \quad (13)$$

$$-\lambda\left.\frac{dt}{dx}\right|_{x=\delta}+\alpha(t_c-t|_{x=\delta})=\rho\cdot q_2'; \quad (14)$$

$$a_{m2}\rho_0\left.\frac{du}{dx}\right|_{x=\delta}+a_{m2}\rho_0\delta_2\left.\frac{dt}{dx}\right|_{x=\delta}=-q_2'. \quad (15)$$

Taking into account that

$$a=\frac{\lambda}{c_v\rho_0}.$$

Differential equation (11) takes the form

$$\frac{d^2t}{dx^2}=-\frac{Q_v}{\lambda}. \quad (16)$$

The solution of differential equation (16) is a successive double integration

$$\frac{d}{dx}\left(\frac{dt}{dx}\right)=-\frac{Q_v}{\lambda};\quad d\left(\frac{dt}{dx}\right)=-\frac{Q_v}{\lambda}dx;$$

$$\int d\left(\frac{dt}{dx}\right)=-\frac{Q_v}{\lambda}\int dx;\quad \frac{dt}{dx}=-\frac{Q_v}{\lambda}x+C_1;$$

$$dt=\left(-\frac{Q_v}{\lambda}x+C_1\right)dx;\qquad \int dt=\int\left(-\frac{Q_v}{\lambda}x+C_1\right)dx;$$

$$t=-\frac{Q_v}{\lambda}\int x dx+C_1\int dx;\qquad t=-\frac{Q_v}{\lambda}\frac{x^2}{2}+C_1x+C_2, \quad (17)$$

where C_1, C_2 are arbitrary constants.

To find arbitrary constants we use boundary conditions. The condition of symmetry (13) after substitution (17) gives the equation

$$\left.\frac{dt}{dx}\right|_{x=0}=-\frac{Q_v}{\lambda}x+C_1|_{x=0}=0,$$

that is

$$C_1=0. \quad (18)$$

Then the formula (17) taking into account (18) takes the form

$$t=-\frac{Q_v}{\lambda}\frac{x^2}{2}+C_2. \quad (19)$$

To find C_2 we use boundary conditions (14 and 15).

First of all, exclude from the boundary conditions (14 and 15) the intensity of evaporation, combining them into one boundary condition

$$-\lambda\left.\frac{d\,t}{dx}\right|_{x=\delta}+\alpha\cdot(t_c-t|_{x=\delta})+a_{m2}\rho_0\rho\left(\left.\frac{d\,u}{dx}\right|_{x=\delta}+\delta_2\left.\frac{d\,t}{dx}\right|_{x=\delta}\right)=0. \quad (20)$$

First, we calculate the derivatives

$$\frac{dt}{dx}=-\frac{Q_v}{\lambda}x;\quad \left.\frac{dt}{dx}\right|_{x=\delta}=-\frac{Q_v\delta}{\lambda}. \quad (21)$$

According to (12), we obtain

$$\frac{d^2u}{dx^2}=-\delta_2\frac{d^2t}{dx^2},$$

that is

$$\frac{du}{dx}=-\delta_2\frac{dt}{dx}+C_3.$$

According to the condition of symmetry (13), the following takes place

$$\left.\frac{du}{dx}\right|_{x=0}=-\delta_2\left.\frac{dt}{dx}\right|_{x=0}+C_3,$$

that is

$$0=0+C_3,\quad C_3=0.$$

Thus, there is an equation

$$\frac{du}{dx}=-\delta_2\frac{dt}{dx}. \quad (22)$$

At the boundary (at x = δ) the following is fulfilled

$$a_{m2}\rho_0\left(\left.\frac{du}{dx}\right|_{x=\delta}+\delta_2\left.\frac{dt}{dx}\right|_{x=\delta}\right)=-q_2'. \quad (23)$$

To find C_2, we substitute (19, 21 and 23) in (20), which will give

$$-\lambda\left.\frac{dt}{dx}\right|_{x=\delta}+\alpha\cdot(t_c-t|_{x=\delta})-\rho\cdot q_2'=0;$$

$$-\lambda\left(-\frac{Q_v\cdot\delta}{\lambda}\right)+\alpha\left(t_c+\frac{Q_v}{\lambda}\frac{\delta^2}{2}-C_2\right)-\rho\cdot q_2'=0;$$

$$C_2=t_c+\frac{\delta^2}{2\lambda}Q_v+\frac{\delta}{\alpha}Q_v-\frac{1}{\alpha}\rho\cdot q_2'. \quad (24)$$

Substituting (24) into (19), we find the temperature distribution over the thickness of the plate

$$t=-\frac{Q_v}{\lambda}\frac{x^2}{2}+t_c+\frac{Q_v}{\lambda}\frac{\delta^2}{2}+\frac{\delta}{\alpha}Q_v-\frac{1}{\alpha}\rho\cdot q_2',$$

or

$$t=-\frac{Q_v}{2\lambda}(x^2-\delta^2)+t_c+\frac{\delta}{\alpha}Q_v-\frac{1}{\alpha}\rho\cdot q_2'. \quad (25)$$

By integrating (22), we find

$$\int_{u_0}^{u}du=-\delta_2\int_{t_0}^{t}dt;$$

$$u-u_0=-\delta_2\cdot(t-t_0). \quad (26)$$

Taking into account (25, 26), the distribution of moisture content in the thickness of the plate is according to the formula

$$u=u_0+\delta_2\left(\frac{Q_v}{2\lambda}(x^2-\delta^2)-t_c+t_0-\frac{\delta}{\alpha}Q_v-\frac{1}{\alpha}\rho\cdot q_2'\right). \quad (27)$$

According to (25), the maximum temperature is reached in the center of the plate (at x = 0)

$$t_{\max}=t(x=0)=\frac{Q_v}{2\lambda}\delta^2+\frac{\delta}{\alpha}Q_v+t_c-\frac{1}{\alpha}\rho\cdot q_2'. \quad (28)$$

The surface of the plate (at x = δ) will be the minimum value of temperature

$$t_{\min}=t(x=\delta)=t_c+\frac{\delta}{\alpha}Q_v-\frac{1}{\alpha}\rho\cdot q_2'. \quad (29)$$

The average plate temperature can be found by integrating the plate thickness

$$\overline{t} = \frac{1}{2\delta}\int_{-\delta}^{\delta}\left(-\frac{Q_v}{2\lambda}(x^2-\delta^2)+t_c+\frac{\delta}{\alpha}Q_v-\frac{1}{\alpha}\rho\cdot q_2'\right)dx. \quad (30)$$

Calculating the integral, sequentially, using the Newton-Leibniz formula, we consistently find

$$\overline{t} = \frac{1}{2\delta}\int_{-\delta}^{\delta}\left(-\frac{Q_v}{2\lambda}(x^2-\delta^2)+t_c+\frac{\delta}{\alpha}Q_v-\frac{1}{\alpha}\rho\cdot q_2'\right)dx;$$

$$\overline{t} = -\frac{Q_v}{2\lambda\delta}\int_{0}^{\delta}(x^2-\delta^2)dx+\frac{1}{\delta}\int_{0}^{\delta}\left(t_c+\frac{\delta}{\alpha}Q_v-\frac{1}{\alpha}\rho\cdot q_2'\right)dx;$$

$$\overline{t} = -\frac{Q_v}{2\lambda\delta}\left(\frac{1}{3}x^3-\delta^2x\right)\Bigg|_0^{\delta}+\frac{1}{\delta}\left(t_c+\frac{\delta}{\alpha}Q_v-\frac{1}{\alpha}\rho\cdot q_2'\right)x\Big|_0^{\delta};$$

$$\overline{t} = -\frac{Q_v}{2\lambda\delta}\left(\frac{1}{3}\delta^3-\delta^3\right)+\frac{1}{\delta}\left(t_c+\frac{\delta}{\alpha}Q_v-\frac{1}{\alpha}\rho\cdot q_2'\right)\delta;$$

$$\overline{t} = \frac{Q_v\delta^2}{3\lambda}+\frac{\delta}{\alpha}Q_v-\frac{1}{\alpha}\rho\cdot q_2'+t_c;$$

$$\overline{t} = Q_v\delta\left(\frac{\delta}{3\lambda}+\frac{1}{\alpha}\right)-\frac{1}{\alpha}\rho\cdot q_2'+t_c. \quad (31)$$

In turn, according to (27), the minimum moisture content is reached in the center of the plate (at $x = 0$)

$$u_{\min} = u(x=0) = u_0-\delta_2\left(Q_v\delta\left(\frac{\delta}{2\lambda}+\frac{1}{\alpha}\right)-\frac{1}{\alpha}\rho\cdot q_2'+t_c-t_0\right). \quad (32)$$

The maximum moisture content takes place on the surface of the plate (at $x = \delta$)

$$u_{\max} = u(x=\delta) = u_0-\delta_2\left(t_c-t_0+\frac{\delta}{\alpha}Q_v-\frac{1}{\alpha}\rho\cdot q_2'\right). \quad (33)$$

The average moisture content is found by integrating (27) the thickness of the plate

$$\overline{u} = \frac{1}{2\delta}\int_{-\delta}^{\delta}\left(u_0+\delta_2\left(\frac{Q_v}{2\lambda}(x^2-\delta^2)-t_c-\frac{1}{\alpha}\rho q_2'+t_0-\frac{\delta}{\alpha}Q_v\right)\right)dx. \quad (34)$$

Integrating (34) using the Newton-Leibniz formula, we consistently find

$$\overline{u} = \frac{1}{\delta}\int_{0}^{\delta}\left(u_0+\delta_2\left(\frac{Q_v}{2\lambda}(x^2-\delta^2)-t_c-\frac{1}{\alpha}\rho\cdot q_2'+t_0-\frac{\delta}{\alpha}Q_v\right)\right)dx;$$

$$\overline{u} = \delta_2\frac{Q_v}{2\lambda\delta}\left(\frac{1}{3}x^3-\delta^2x\right)\Bigg|_0^{\delta}+\frac{1}{\delta}\times$$
$$\times\left(u_0+\delta_2\left(-t_c-\frac{1}{\alpha}\rho\cdot q_2'+t_0-\frac{\delta}{\alpha}Q_v\right)\right)x\Big|_0^{\delta};$$

$$\overline{u} = -\delta_2\frac{Q_v\delta^2}{3\lambda}+u_0+\delta_2\left(-t_c-\frac{1}{\alpha}\rho\cdot q_2'+t_0-\frac{\delta}{\alpha}Q_v\right);$$

$$\overline{u} = u_0-\delta_2\left(Q_v\delta\left(\frac{\delta}{3\lambda}+\frac{1}{\alpha}\right)+t_c+\frac{1}{\alpha}\rho\cdot q_2'-t_0\right). \quad (35)$$

For further research with the help of the obtained formulas it is expedient to bring them to a dimensionless form. For this purpose, we can use the theory of similarity and dimensional analysis. The application of the π-theorem allows reducing the number of variables by combining them into dimensionless complexes and similarity criteria.

With respect to formula (25), the theory of similarity and dimensional analysis allows us to present it in such a dimensionless form

$$t = -\frac{Q_v}{2\lambda}(x^2-\delta^2)+t_c+\frac{\delta}{\alpha}Q_v-\frac{1}{\alpha}\rho\cdot q_2';$$

$$\theta = \frac{1}{2}(1-X^2)+\frac{1}{Bi}-K; \quad (36)$$

at

$$\theta = \frac{\lambda\cdot(t-t_c)}{Q_v\cdot\delta^2};\quad X = \frac{x}{\delta};$$

$$Bi \equiv \frac{\alpha\cdot\delta}{\lambda};\quad K = \frac{\lambda\cdot\rho\cdot q_2'}{\alpha\cdot Q_v\delta^2}.$$

Formula (36) determines the dependence of the dimensionless temperature on the dimensionless distance X and the criteria of Bio Bi and K. It should be noted that criterion K evaluates the ratio of evaporation intensities and internal heat sources.

After solving the system of differential equations (8) with the accepted boundary conditions (13–15), we obtain dependences that can be used to calculate the change in temperature (37) and moisture content (38) of the layer of fine material depending on the time of electric current.

$$t(x,\tau) = t_0+\left(t_c-t_0-\frac{\rho\cdot q_2'}{\alpha}\right)\times$$
$$\times\left(1-2\sum_{n=1}^{\infty}\frac{\sin\mu_n}{\mu_n+\cos\mu_n\sin\mu_n}\cos\left(\mu_n\frac{x}{\delta}\right)e^{-\mu_n^2\frac{a\cdot\tau}{\delta^2}}\right)-$$
$$-\frac{Q_v}{c_v\rho_0}\left[\frac{1}{2a}\left(x^2-\delta^2\left(1+\frac{2}{Bi}\right)\right)+\right. \quad (37)$$
$$\left.+\frac{2\delta^2}{a}\sum_{n=1}^{\infty}\frac{\sin\mu_n}{\mu_n^2(\mu_n+\cos\mu_n\sin\mu_n)}\cos\left(\mu_n\frac{x}{\delta}\right)e^{-\mu_n^2\frac{a\cdot\tau}{\delta^2}}\right];$$

$$u(x,\tau) = u_0-\delta_2\frac{\sqrt{a_{m2}}}{a_{m2}-a}\left(t_c-t_0-\frac{\rho\cdot q_2'}{\alpha}\right)\times$$
$$\times 2\sum_{n=1}^{\infty}\left(\sqrt{a}\frac{\sin\mu_n}{\sin\sqrt{\frac{a}{a_{m2}}}\mu_n}\cos\mu_n\sqrt{\frac{a}{a_{m2}}}\frac{x}{\delta}-\sqrt{a_{m2}}\cos\mu_n\frac{x}{\delta}\right)\times$$
$$\times\frac{\sin\mu_n}{\mu_n+\cos\mu_n\sin\mu_n}e^{-\mu_n^2\frac{a}{\delta^2}\tau}-\delta_2\frac{\sqrt{a_{m2}}}{a_{m2}-a}\frac{Q_v}{c_v\rho_0}\times$$
$$\times\left[\frac{a-a_{m2}}{2a\sqrt{a_{m2}}}\left(x^2-\frac{\delta^2}{2}\right)+\right. \quad (38)$$
$$+\frac{\delta^2}{a}2\sum_{n=1}^{\infty}\left(\sqrt{a}\frac{\sin\mu_n}{\sin\sqrt{\frac{a}{a_{m2}}}\mu_n}\cos\mu_n\sqrt{\frac{a}{a_{m2}}}\frac{x}{\delta}-\sqrt{a_{m2}}\cos\mu_n\frac{x}{\delta}\right)\times$$
$$\left.\times\frac{\sin\mu_n}{\mu_n^2(\mu_n+\cos\mu_n\sin\mu_n)}e^{-\mu_n^2\frac{a}{\delta^2}\tau}\right]-$$
$$-\frac{q_2'}{\sqrt{a_{m2}}\rho_0}\left[\frac{\sqrt{a_{m2}}}{\delta}\tau+\frac{1}{2\delta\sqrt{a_{m2}}}\left(x^2-\frac{\delta^2}{2}\right)\right].$$

Results. To verify the adequacy of the model, experiments were performed on a specially designed laboratory installation (Fig. 2).

Dependence testing (37, 38) was performed using Mathcad software for each series of experimental studies. For laboratory studies on the drying process by direct exposure to electric current of fine enrichment products and elucidation of the basic laws of this process assembled installation (Fig. 2) consisting of two galvanized electrodes 7 size 70 × 100 mm, placed in a textolite cup, laboratory autotransformer, relay time and control – measuring devices: thermal imager, wattmeter, ammeter, meter. A portion of the source material is loaded between the electrodes and sealed by a load 3 weighing 12 kg, through a rod with a piston 6 made in the form of a dielectric grating. The distance between the electrodes is regulated by the selection of interlayer.

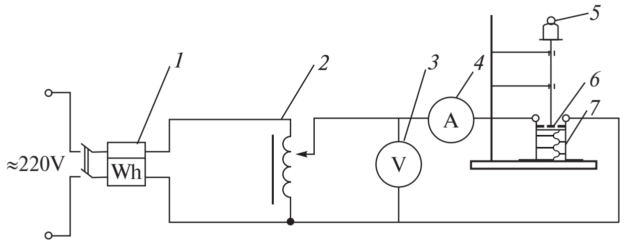

Fig. 2. Scheme of a laboratory installation for research on process of drying fine dispersed materials by direct influence of an electric current:

1 – counter; 2 – laboratory autotransformer; 3 – wattmeter; 4 – ammeter; 5 – load; 6 – rod with piston; 7 – electrode plates

Sand with a particle size of 0.1–0.2 mm was selected for the experiments, the initial moisture varied from 14 to 18 % with a step of 1 % and the voltage varied from 150 to 250 V with a step of 50 V.

The dependences of the moisture content of the material on the time of electric current and temperature change on the drying time are designed (for sand with a moisture content of 14, 16, 18 % at a voltage of 250 V) (Figs. 3, 4).

As can be seen from Fig. 3, the final moisture curve is asymptomatic to the horizontal axis; it is explained by a decrease in the rate of moisture removal due to a decrease in the electrical conductivity of the material with a decrease in its moisture, and as a consequence of a decrease in the power of internal heat sources.

The temperature curve (Fig. 4) is parabolic. This can be explained by the fact that in the first period of time (0–180 s.) the material is heated, in the second period (180–300 s.) There is vaporization with heat supply due to which the temperature of the material decreases.

The average deviation of the calculated and experimental data for the entire range u and Δ u is about 5 %, the maximum does not exceed 12 % so the mathematical model can be considered adequate.

The graph (Fig. 5) shows that the specific energy consumption using this method of drying averages 0.8 kWh/kg, which confirms the efficiency of its use.

Conclusions.

1. As a result of mathematical modeling, a new solution is elaborated to the current scientific problem of establishing patterns of drying of fine materials by direct influence of alternating electric current in the form of temperature and moisture content on the drying time and spatial coordinates, whose feature is simultaneous accounting of thermal and diffusion processes in the material.

2. Laboratory studies have confirmed the adequacy of the obtained dependences, that the average deviation of the calculated and experimental data for the entire range u and Δ u is about 5 %, the maximum does not exceed 12 %.

3. The received dependences are used at establishment of rational parameters of drying and development of a technique for calculation of the industrial drying installation.

4. In the future, the results of the research will be used in the development of a combined method of dehydration.

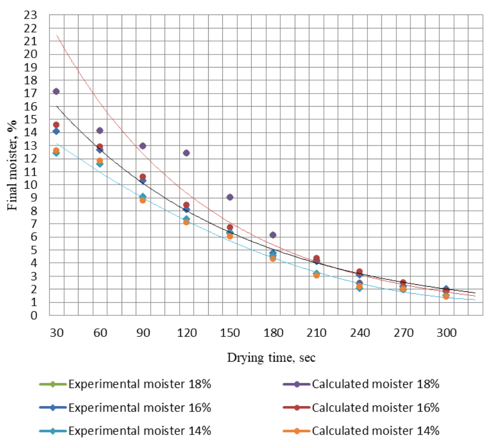

Fig. 3. Graph of the dependence of moisture on the material from the time of electric current

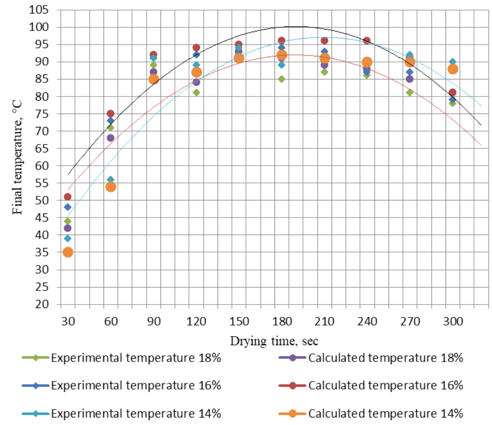

Fig. 4. Graph of temperature change from drying time

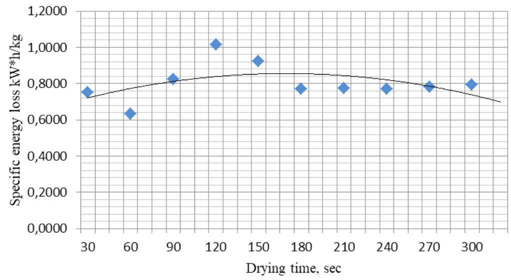

Fig. 5. Graph of the specific energy loss for the removal of 1 kg of moisture (initial moisture – 18 %, voltage 250 V)

References.

1. Kyrychenko, E., Samusya, V., Kyrychenko, V., & Antonenko, A. (2015). Thermodynamics of Multiphase Flows in Relation to The Calculation of Deep-Water Hydraulic Hoisting. *New Developments in Mining Engineering: Theoretical and Practical Solutions of Mineral Resources Mining*, 305-311. https://doi.org/10.1201/b19901-54.

2. Souza Pinto, T. C., Souza, A. S., Batista, J. N. M., Sarkis, A. M., Leal Filho, L. S., Pádua, T. F., & Béttega, R. (2020). Characterization and drying kinetics of iron ore pellet feed and sinter feed. *Drying Technology*. https://doi.org/10.1080/07373937.2020.1747073.

3. Hao Zhou, Zhenya Lai, Laiquan Lv, Hao Fang, Hanxiao Meng, Mingxi Zhou, & Kefa Cen (2020). Improvement in the permeability of sintering beds by drying treatment after granulating sinter raw materials containing concentrates. *Advanced Powder Technology, 31*(8), 3297-3306. https://doi.org/10.1016/j.apt.2020.06.017.

4. Zamansky, I. B., Cohen, V. Kh., Rodionova, O. P., & Trochina, G. A. (1987). *Installation for high-frequency drying of dielectric materials*. USSR Patent No. 3894221.

5. Pievsky, I. M., & Milstein. I. Z. (1983). *Method of drying products from capillary-porous materials*. USSR Patent No. 363363112.

6. Antipov, S. T., Kazartsev, D. A., & Pavlovsky, M. Yu. (2003). *Microwave-installation for loose materials drying*. Russian Federation Patent No. 2255434. State Educational Institution Voronezh State Technological Academy.

7. Yur'eva, B., Dudkob, V., & Biakova, M. (2018). *Application of Ecologically Clean Technology to Drying of Iron-Ore Pellets*. Trans Tech Publications Ltd, Switzerland. https://doi.org/10.4028/www.scientific.net/SSP.284.828.

8. Zamitskyi, O. V., Bondar, N. V., & Kradozhon, S. O. (2019). Innovative technologies in the process of drying fine materials. *Visnyk Kryvorizkoho Natsionalnoho Universytetu, 48*, 83-88. https://doi.org/10.31721/2306-5451-2019-1-48-83-88.

9. Efimov, V. N., Sidorenko, Yu. A., Agafonov, D. A., & Yeltsin, S. I. (1999). *Method of wet bulk materials drying*. Russian Federation Patent No. 2143655. JSC "Krasnoyarsk Non-Ferrous Metals Plant".
10. Antipov, S. T., Vizir, D. M., Shakhov, S. V., & Zhuchkov, A. V. (2012). Mathematical model of the process of thermal regeneration of diatomaceous earth. *Vestnik Voronezskogo gosudarstvennogo universiteta inzenernyh tehnologij*, (1), 62-67.

Математична модель процесу сушки тонкодисперсних матеріалів впливом змінного електричного струму

О. В. Замицький, *Н. О. Голівер*, *Н. В. Бондар*, *С. О. Крадожон*

Криворізький національний університет, м. Кривий Ріг, Україна, e-mail: kradozhon.so@knu.edu.ua

Мета. Встановлення закономірностей процесу сушки тонкодисперсних матеріалів прямим впливом електричного струму для визначення раціональних параметрів процесу.

Методика. У роботі використовувалися теоретичні, аналітичні, емпіричні, експериментальні методи й методи математичної статистики. Проведено математичне моделювання процесу, що протікає при сушці тонкодисперсних матеріалів прямим впливом змінного струму.

Результати. У результаті дослідження на основі фізичних уявлень про процес сушіння капілярно-пористого матеріалу побудована математична модель, що зв'язує за допомогою рівнянь математичної фізики температуру й вологовміст у пластині з капілярно-пористого матеріалу.

Наукова новизна. Уперше одержані залежності температури та вмісту вологи від часу й просторової координати при сушки шляхом пропускання електричного струму через шар вологого капілярно-пористого матеріалу, особливістю яких є одночасний облік протікання теплових і дифузійних процесів у матеріалі, що дозволяє підвищити точність розрахунків і встановити раціональні параметри сушки.

Практична значимість. Отримані залежності використані при розробці методики розрахунку та конструкції промислової сушильної установки.

Ключові слова: *математична модель, тонкодисперсні матеріали, сушка*

Recommended for the publication by M. V. Kiyanovskyi, Doctor of Technical Sciences. The manuscript was submitted 28.12.20.