UDC 622.775
DOI: 10.15587/1729-4061.2021.224939

ORE CRUSHING IN THE HIGH-PRESSURE ROLLER-PRESS AS A MODELLING OBJECT UNDER STOCHASTIC PROPERTIES OF FEED MATERIALS

T. Oliinyk
Doctor of Technical Sciences, Professor,
Head of Department*
E-mail: taoliynik@knu.edu.ua
P. Nikolaienko
Postgraduate Student*
E-mail: pavlo.nikolaienko@gmail.com
K. Nikolaienko
PhD, Associate Professor*
E-mail: asprof.nikolaienko@gmail.com
M. Oliinyk
PhD, Senior Lecturer*
E-mail: maksym.oliinyk@knu.edu.ua
*Department of Mineral Processing and Chemistry
Kryvyi Rih National University
Vitaliya Matusevycha str., 11, Kryvyi Rih, Ukraine, 50027

The results of improving ore crushing in a high-pressure roller-press are presented. Application of a roller-press enables higher crushing efficiency due to both power saving and reduction of sizes of ore crush products to release mineral aggregates. Ore disintegration by compressive strain prevails among currently applied crushing methods. Disintegration occurs not only due to the compressive, but also to the shear strain. Considering smaller power consumption of the shear strain than that of the compressive strain, it is concluded that roller-press application is quite efficient.

Simulation of crushing by using the Bond law frequently applied in practice is under consideration. It is essential to consider the stochasticity of the ore flow to be crushed. Presentation of this flow as a random figure by transforming it by the Bond crushing law results in a probabilistic characteristic of the crushing result. This characteristic enables finding properties of the crush product and probabilistic formulation of the problem of improving the crushing process by setting a relevant functional. To apply the results obtained to practical uses, the crushing process is simulated. The theoretical results are confirmed by setting the stochastic properties of the input ore flow by means of Rosen-Rammler's law followed by statistical substantiation of the conducted calculations in Mathcad. After stimulation and considering stochastic properties of the feed ore flow, the solution of the optimal stabilization problem reveals that stabilization is achieved, while dispersion in relation to the stabilization goal reduces sharply almost five-fold

Keywords: crushing, roller-press, ore beneficiation, distribution function, simulation, deformation, stochasticity

Received date 29.10.2020
Accepted date 29.01.2021
Published date 19.02.2021

1. Introduction

Modern requirements to the end quality of beneficiated ore indicate the necessity to apply innovative methods to its treatment. This results in considering peculiarities of ore treatment on each of beneficiation stages. The efficiency of ore beneficiation depends on the degree of minerals release in ore treatment with their simultaneous minimal crushing [1]. In spite of great achievements in the field of ore treatment, very few technological solutions meet one of the principal requirements of optimal beneficiation – no extra crushing. It is conditioned by poor understanding of the selective disintegration mechanism, uninvestigated connections of ore properties, minerals, disintegration conditions and grinding indices. At initial disintegration stages, textual characteristics of ores and microscopic physical and mechanical properties of rocks play a major role in forming structural elements of mineral release. While reducing a lump size, the role of structural characteristics of ore and properties of individual minerals and aggregates is becoming more important. This assumes the increased number of marketable flows and «fine» adjustment of relevant stages of ore treatment including both disintegration and separation.

Analysis of ore treatment as the initial stage of beneficiation indicates that crushing should be paid particular attention to. It is associated with reduced specific energy consumption as compared with grinding as well as with finer crushing products in modern roller presses, enabling greater efficiency of subsequent stages of beneficiation. For this reason, to improve the efficiency of ore treatment, most of disintegration operations should be focused on crushing.

It is evident that researches and practical uses are of particular topicality here in current conditions of ore beneficiation as requirements to characteristics of end products of processing are rising.

2. Literature review and problem statement

[2] investigates the issues of developing determined models of crushing. It is revealed that after considering the physical essence and analyzing the suggested theories, it is

possible to build systems of controlling crushing cycles. At the same time, the issues of assessing the efficiency of these control systems remain unsolved. It can be explained by the absence of reliable tools for collecting and processing the data on crushing processes [3] investigates the issues of identifying specific power expenses for fixed-bed crushing in cone crushers considering regularities of particle disintegration. The algorithm of calculating specific power consumption for fixed-bed crushing in cone crushers is developed. With that, the research results are tested in cone crushers only to obtain cubical crushed stone. The authors of [4] deal with the stochastic simulation of disintegrating polycrystalline particles resulted from ore destruction in crushing operations. They indicate that introduction of stochastic functions of size reduction and classification of particles into the kinetic equation of disintegration and release enables forecasting technological indices of ore material treatment. Meanwhile, the described simulation method requires a greater volume of a priori information followed up by its processing.

Scientific papers describing ore fixed-bed treatment in roller-presses are of particular interest. It is stated that this method enables accomplishing several crushing and grinding stages in a single apparatus and reducing power consumption simultaneously. Besides, control over the disintegration process in this technology will increase the release of valuable components due to reducing their overgrinding. All the researchers of these issues indicate that the granulometric composition of the crushing process is the basic characteristic of a roller-press functioning. Yet, there is no single opinion as to simulating this very characteristic [5] considers simulation of curves of granulometric composition of crush products and notes that crushing results can be controlled by selecting a particular value of pressure on the ore bed. [6] applies the distribution function of disintegration of ore lumps that is associated with the characteristic of the initial material to assessing ore sizes [7] also indicates that besides the ore disintegration distribution, the clearance between rollers has a great impact on the size of crush products. The authors of [8] note that the granulometric composition of products is influenced by the location of rollers. [9] demonstrates that the models based on the structure similar to the work index should depend on the same size to represent the distribution of sizes as a whole. Thus, these models require more empirical coefficients introduced.

[10] mentions the main problem hindering effective implementation of control over fixed-bed disintegration in compression which involves understudied mechanisms of its course. Thus, because of the absence of reliable theoretical and empirical methods, it is suggested to use computer technologies of engineering analysis based on matrix mathematical methods of numerical solution. In particular, the method of finite elements applied to determining the stress-strain state of ores is of great efficiency here. It should be accentuated that the given way of studying the stress-strain state aimed at breaking ore is both costly and time-consuming.

[11] applies a mathematical model with finite contact elements of ore interaction to assessing the energy state of ore under fixed-bed disintegration, allowing determining the granulometric composition of crushed ore. The experiments conducted in industrial conditions confirm the efficiency of the synthesized mathematical model. Meanwhile, analysis of the above work indicates the complex character of implementing the described methods [12] discusses the issues of the causes of reduced power consumption in ore disintegration in roller-presses. This is mainly caused by the insufficient rate of investigation of causes of reduced power consumption in ore disintegration in roller-presses. Mathematical simulation by finite element analysis reveals that power is saved when ore is disintegrated. The data presented in [13] dealing with selective disintegration of mineral and technological raw materials are essential. In particular, the role of stress factors in selective disintegration is illustrated by ores of ferrous metals. New results of experimental and theoretical researches enable the expansion of the concept of ore selective disintegration [14, 15]. Yet, the issues of applying mathematical simulation to selective disintegration of ores remain under-investigated.

The mentioned literature sources enable concluding that currently a solution to the problem of ore selective disintegration by high-pressure roller-presses is essential. Mathematical simulation of crushing processes in roller presses will allow improving ore selective disintegration avoiding the trial-error method.

3. The aim and objectives of the study

The research aims to simulate the ore crushing process in a high-pressure roller-press under stochastic properties of feed materials.

This will enable synthesizing an algorithm of controlling the crushing process to achieve optimal stabilization of modes as for the set diameter of crush products under stochastic properties of feed materials.

To achieve the aim set, there are the following objectives to be accomplished:

– developing a mathematical model of ore crushing considering stochastic properties of feed materials;

– synthesizing the algorithm of controlling the crushing process to stabilize the mode as for the set average diameter of crush products under stochastic properties of feed materials.

4. Materials and methods of researching ore fixed-bed crushing

Crushing experiments were conducted in the HPGR 500/15-1000 roller-press using 40-0 mm oxidized iron ore from the Kryvyi Rih iron ore basin. The feed ore and the crush product were subject to sieve analysis for narrow size classes with 2 mm-intervals. Each size class was weighted and its yield was calculated considering the total mass of the material, enabling the determination of granulometric characteristics of the product.

5. Development of the mathematical model of ore fixed-bed crushing considering stochastic properties of feed materials

Simulation of the crushing process is based on the known laws of crushing. Mechanical properties of ore due to their ambiguity cannot be used for obtaining strict calculation equations that determine crushing. Therefore, the current practice is based on the ratios resulted from a great amount of empirical experience canonized as laws of crushing [15].

The laws of crushing are interpreted as a dependency between the power spent on crushing and the size of a crush product. The most applicable expression of empirical dependency characterizing power consumption for size reduction is presented in the differential form as Bond's crushing law:

$$dE = -k\frac{dx}{x^{1.5}}, \quad (1)$$

where E is the specific energy conveyed to a unit of the disintegrated body and required to increase the energy of the newly created surface, J/m^3; k is the proportionality coefficient; x is the average diameter of grains, mm.

To find work in ore crushing in the set range of product sizes, one should solve the Cauchy problem [17], which includes the differential equation (1) and the initial condition determining the size of the feed ore lump at the beginning of crushing:

$$E|_{x=d_0} = 0, \quad (2)$$

where d_0 is the diameter of the feed ore lump, mm.

The general solution of differential equation (10) as the one with separable variables looks like:

$$E = \frac{k}{\sqrt{x}} + C, \quad (3)$$

where C is a constant.

Considering condition (2), we find the value of the constant:

$$C = -\frac{k}{\sqrt{d_0}}. \quad (4)$$

Substituting (4) into (3), we find the solution of the Cauchy problem (1), (2):

$$E = k\left(\frac{1}{\sqrt{x}} - \frac{1}{\sqrt{d_0}}\right). \quad (5)$$

Equation (5) allows assessing energy consumption for reducing the averaged diameter of the crushed ore lump from the original diameter to the reduced one. After finishing the crushing process, the average energy consumption will make:

$$E_1 = k\left(\frac{1}{\sqrt{d_1}} - \frac{1}{\sqrt{d_0}}\right), \quad (6)$$

where d_1 is the diameter of the crush product, mm.

It is worth mentioning that the averaged diameter of a feed ore lump and energy consumption are input variables while the averaged diameter of the crush product is the output variable of the crushing process model. Considering the above, equation (6) should be reduced to:

$$d_1 = \frac{d_0}{\left(\frac{E_1}{k}\sqrt{d_0} + 1\right)^2}. \quad (7)$$

Analysis of equation (7) reveals that it contains a single parameter k.

It should be underlined that the mathematical model of the crushing process (7) is noted for the impact of the input variable – the size of the feed ore. Input impact (sizes of ore lumps) is a random value with some distribution law. That is why, after crushing the output variable in the form of crushed lumps of the same ore is also a random variable, yet with another distribution law. It is natural to wonder what the characteristic of the crush product as a random value is like.

Let the size of the feed ore as a continuous random value be set by a differential distribution function. It is necessary to find a differential distribution function of the crush product sizes, which is a random value determined by formula (7). As function (7) is monotonically increasing and differential, the inverse function exists and is also monotonically increasing and differential. At that, the formula determining this inverse function according to (6) looks like:

$$d_0 = \frac{d_1}{\left(1 - \frac{E_1}{k}\sqrt{d_1}\right)^2}. \quad (8)$$

If, on the axis of the output variable, we set an interval and reflect it by function (8) onto the axis of the input variable, we obtain some interval as well.

Events of hitting these intervals are causally determined as they are functionally connected and equal, i. e.:

$$P(d_1 < D_1 < d_1 + \Delta d_1) = P(d_0 < D_0 < d_0 + \Delta d_0), \quad (9)$$

where $(d_0<D_0<d_0+\Delta d_0)$, $(d_1<D_1<d_1+\Delta d_1)$ are input and output intervals.

According to the definition of a differential distribution function, we have:

$$g(d_1) = \lim_{\Delta d_1 \to 0} \frac{P(d_1 < D_1 < d_1 + \Delta d_1)}{\Delta d_1}. \quad (10)$$

Considering (9), formula (10) can be presented as:

$$g(d_1) = \lim_{\Delta d_1 \to 0} \frac{P(d_0 < D_0 < d_0 + \Delta d_0)}{\Delta d_1}. \quad (11)$$

Taking into account the fact that (8) is a monotonically increasing and differential, we can write down the expression to increase it:

$$\Delta d_0 = d_0'(d_1)\Delta d_1. \quad (12)$$

As

$$d_0'(d_1) = \left|1 - \frac{E_1}{k}\sqrt{d_1}\right|^{-3},$$

formula (12) looks like:

$$\Delta d_0 = \left|1 - \frac{E_1}{k}\sqrt{d_1}\right|^{-3} \Delta d_1. \quad (13)$$

Substituting (13) into (11), we find:

$$g(d_1) = \lim_{\Delta d_1 \to 0} \frac{P(d_0 < D_0 < d_0 + \Delta d_0)}{\Delta d_0} \frac{1}{\left|1 - \frac{E_1}{k}\sqrt{d_1}\right|^3}. \quad (14)$$

According to the definition of a differential distribution function of the feed ore sizes as a random value, we have the equation:

$$f(d_0) = \lim_{\Delta d_0 \to 0} \frac{P(d_0 < D_0 < d_0 + \Delta d_0)}{\Delta d_0}. \quad (15)$$

Considering continuity of function (8), formula (14) considering (15) looks like:

$$g(d_1)=f(d_0)\frac{1}{\left|1-\frac{E_1}{k}\sqrt{d_1}\right|^3}. \qquad (16)$$

Finally, using formula (8), we obtain the final expression for the differential distribution function of the crush product sizes:

$$g(d_1)=f\left(\frac{d_1}{\left(1-\frac{E_1}{k}\sqrt{d_1}\right)^2}\right)\frac{1}{\left|1-\frac{E_1}{k}\sqrt{d_1}\right|^3}. \qquad (17)$$

Knowing the differential distribution function of the feed ore size, the integral distribution function is written as:

$$F(d_0)=\int_0^{d_0} f(x)\mathrm{d}x. \qquad (18)$$

In its turn, the integral distribution function of the crush product size will look like:

$$G(d_1)=\int_0^{d_1} g(y)\mathrm{d}y. \qquad (19)$$

Considering (17), formula (19) is written as:

$$G(d_1)=\int_0^{d_1} f\left(\frac{y}{\left(1-\frac{E_1}{k}\sqrt{y}\right)^2}\right)\frac{1}{\left|1-\frac{E_1}{k}\sqrt{y}\right|^3}\mathrm{d}y. \qquad (20)$$

By transforming integral (20), we can show that the integral distribution function of the crush product size is associated with the integral distribution function of the feed ore size by the equation:

$$G(d_1)=F\left(\frac{d_1}{\left(1-\frac{E_1}{k}d_1\right)^2}\right). \qquad (21)$$

According to (21), the problem of identifying parameters of the crushing model k is solved by reducing the functional [18]:

$$\sum_{j=1}^{M}\sum_{i=1}^{N}\left(G^*(d_{1i},E_{1j})-F\left(\frac{d_{1i}}{\left(1-\frac{E_{1j}}{k}d_{1i}\right)^2}\right)\right)^2 \to \min_k, \qquad (22)$$

where $G^*(d_{1i}, E_{1j})$ is the value of the sieve analysis corresponding to the i-th size of the sieve for the j-th value of energy consumption for crushing; N is the number of sieves; M is the number of values of energy consumption for crushing.

Calculated differential distribution functions of the feed ore and crush product sizes allow calculating numerical characteristics of these variables. Mathematical expectation and dispersion of sizes of the feed ore are found by the corresponding formulae:

$$M[D_0]=\int_{\underline{d_0}}^{\overline{d_0}} xf(x)\mathrm{d}x,$$

$$D[D_0]=\int_{\underline{d_0}}^{\overline{d_0}} (x-M[D_0])^2 f(x)\mathrm{d}x, \qquad (23)$$

where $\underline{d_0}$, $\overline{d_0}$ is the smallest and the largest average diameters of the feed ore lumps.

Similarly, the mathematical expectation and dispersion of sizes of the crush product are found:

$$M[D_1]=\int_{\underline{d_1}}^{\overline{d_1}} xg(x)\mathrm{d}x,$$

$$D[D_1]=\int_{\underline{d_1}}^{\overline{d_1}} (x-M[D_1])^2 g(x)\mathrm{d}x, \qquad (24)$$

where $\underline{d_1}$, $\overline{d_1}$ is the smallest and the largest average diameters of the crushing product lump.

Thus, formulae (23) and (24) allow expressing the most essential peculiarities of the studied distributions in a concise form. The first formula of (24) determines the average value around which probable values of sizes of the crush product as a random value are grouped. In its turn, the second formula of (24) determines the degree of dispersion of values of the crush product sizes with regard to its average value.

Mathematical description of crushing the layer of particles under pressure is obligatory for building the system controlling this process as a complex object.

At the same time, it is necessary to accentuate the role of the model in the system controlling the technological process of crushing the layer of ore particles.

First of all, it is important to indicate the aim of controlling this process when using a roller-press. While controlling the technological process of crushing, there appears a situation characterized by a triplet:

$$\langle D_0, D_1, d_1^* \rangle, \qquad (25)$$

where d_1^* is the optimal average diameter of the crush product lump.

In the case under consideration, the optimal average diameter of the crush product lump is the control aim.

6. Synthesis of the algorithm of controlling the crushing process to stabilize the mode as for the set average diameter of crush products under stochastic properties of feed materials

In mathematical model (7), the energy impact is a controlling input variable. Thus, the problem of optimal stabilization can be formulated as an extreme problem [19]:

$$\int_{\underline{d_1}}^{\overline{d_1}} (x-d_1^*)^2 g(x)\mathrm{d}x \to \min_{E_l \le E \le E_u},$$

or considering (17):

$$\int_{\frac{\underline{d_0}}{\left(\frac{E_1}{k^*}\sqrt{\underline{d_0}}+1\right)^2}}^{\frac{\overline{d_0}}{\left(\frac{E_1}{k^*}\sqrt{\overline{d_0}}+1\right)^2}} (x-d_1^*)^2 f\left(\frac{x}{\left(1-\frac{E_1}{k^*}\sqrt{x}\right)^2}\right)\frac{1}{\left|1-\frac{E_1}{k^*}\sqrt{x}\right|^3}\mathrm{d}x \to$$

$$\to \min_{E_l \le E \le E_u}, \qquad (26)$$

where E_l, E_u are the lower and the upper boundaries of the controlling impact.

The quality of optimal stabilization of crushing is assessed through the value of deviation of the input variable from the set optimal value and calculated by:

$$\sigma_1=\sqrt{\int\limits_{\frac{\underline{d}_0}{\left(\frac{E_1^*}{k^*}\sqrt{\underline{d}_0}+1\right)^2}}^{\frac{\overline{d_0}}{\left(\frac{E_1^*}{k^*}\sqrt{\overline{d_0}}+1\right)^2}}\left(x-d_1^*\right)^2 f\left(\frac{x}{\left(1-\frac{E_1^*}{k^*}\sqrt{x}\right)^2}\right)\frac{1}{\left|1-\frac{E_1^*}{k^*}\sqrt{x}\right|^3}\mathrm{d}x},\quad(27)$$

where k^* is the optimal value of the identification parameter; E_1^* is the optimal value of the controlling impact.

By analyzing the mean-square value of deviation of the output variable from the set value calculated by (27), we can say whether the control aim is accessible, enabling the conclusion about further actions.

When studying the distribution of sizes of original ore, researchers often use Rosin-Rammler's distribution [20], the integral function of which looks like:

$$F\left(d_0\right)=1-e^{-\left(\frac{d_0}{\Delta}\right)^n},\quad(28)$$

where Δ and n are parameters.

The differential function of this distribution will be a derivative of the integral distribution function (28):

$$f\left(d_0\right)=\frac{n}{\Delta^n}d_0^{n-1}e^{-\left(\frac{d_0}{\Delta}\right)^n}.\quad(29)$$

The mathematical expectation of the feed ore sizes obtained through integration (29) is written as the integral:

$$M\left[D_0\right]=n\int\limits_0^{\infty}\left(\frac{x}{\Delta}\right)^n e^{-\left(\frac{x}{\Delta}\right)^n}\mathrm{d}x=\Delta\cdot\Gamma\left(1+\frac{1}{n}\right),\quad(30)$$

where $\Gamma(1+1/n)$ is the Euler gamma-function [21].

Dispersion of the feed ore sizes of this distribution is also expressed through the Euler gamma-function:

$$D\left[D_0\right]=\int\limits_0^{\infty}\left(x-M\left[D_0\right]\right)^2 f(x)\mathrm{d}x=$$

$$=\Delta^2\cdot\left(\Gamma\left(1+\frac{2}{n}\right)-\Gamma\left(1+\frac{1}{n}\right)^2\right).\quad(31)$$

It should be noted that Δ characterizes the average size of feed ore lumps, while the value n determines the high density of distribution compared with the average size.

In this case, according to (16) and (21), the integral and differential distribution functions of the crush product sizes are written as:

$$G\left(d_1\right)=1-e^{-\left(\frac{d_1}{\Delta\left(1-\frac{E_1}{k}\sqrt{d_1}\right)^2}\right)^n},\quad(32)$$

$$g\left(d_1\right)=\frac{n}{\Delta^n}\left(\frac{d_1}{\left(1-\frac{E_1}{k}\sqrt{d_1}\right)^2}\right)^{n-1}e^{-\left(\frac{d_1}{\Delta\left(1-\frac{E_1}{k}\sqrt{d_1}\right)^2}\right)^n}\frac{1}{\left|1-\frac{E_1}{k}\sqrt{d_1}\right|^3}.\quad(33)$$

To identify a parameter included in the crushing model, it is necessary to solve the problem of reducing the functional:

$$\sum_{j=1}^{M}\sum_{i=1}^{N}\left(G^*\left(d_{1i},E_{1j}\right)-1+e^{-\left(\frac{d_{1i}}{\Delta\left(1-\frac{E_{1j}}{k}\sqrt{d_{1i}}\right)^2}\right)^n}\right)^2\to\min_k,\quad(34)$$

where $G^*\left(d_{1i},E_{1j}\right)$ are the values of sieve analysis corresponding to the i-th sieve size; for the j-th value of energy consumption for crushing; I is the number of sieves; M is the number of values of energy consumption for crushing.

The mathematical expectation of the crush product sizes considering (33) is found by integration:

$$M\left[D_1\right]=\int\limits_0^{\overline{d_1}}e^{-\left(\frac{x}{\Delta\left(1-\sqrt{\frac{x}{\overline{d_1}}}\right)^2}\right)^n}\mathrm{d}x,\quad(35)$$

where $\overline{d_1}=\left(k^*/E_1\right)^2$ is the maximum size of the crush product.

Dispersion of the crush product sizes considering (33) is found by partial integration:

$$D\left[D_1\right]=\int\limits_0^{\overline{d_1}}xe^{-\left(\frac{x}{\Delta\left(1-\sqrt{\frac{x}{\overline{d_1}}}\right)^2}\right)^n}\mathrm{d}x-M\left[D_1\right]^2.\quad(36)$$

For Rosen-Rammler's distribution, the differential distribution function of which is set by (29), the optimal stabilization problem (26) is written as:

$$\int\limits_0^{\left(\frac{k^*}{E_1}\right)^2}\left(x-d_1^*\right)^2\left(x\left(1-\frac{E_1}{k^*}\sqrt{x}\right)^{-2}\right)e^{-\left(\frac{x}{\Delta\cdot\left(1-\frac{E_1}{k^*}\sqrt{x}\right)^2}\right)^n}\frac{1}{\left|1-\frac{E_1}{k^*}\sqrt{x}\right|^3}\mathrm{d}x\to\min_{E_l\le E_1\le E_u}.\quad(37)$$

According to (27), the quality of improving the crushing process is determined by the formula:

$$\sigma_1=\sqrt{\frac{n}{\Delta^n}\int\limits_0^{\left(\frac{k^*}{E_1^*}\right)^2}\left(x-d_1^*\right)^2\left(x\cdot\left(1-\frac{E_1^*}{k^*}\sqrt{x}\right)^{-2}\right)e^{-\left(\frac{x}{\Delta\cdot\left(1-\frac{E_1^*}{k^*}\sqrt{x}\right)^2}\right)^n}\frac{1}{\left|1-\frac{E_1^*}{k^*}\sqrt{x}\right|^3}\mathrm{d}x}.\quad(38)$$

Analysis of the value of deviation of the input variable from the set optimal value calculated by (38) enables assessing the control aim achievement.

According to the initial data, oxidized iron ore with the fractional composition presented in Fig. 1 and Table 1 according to [22] is fed to the roller-press.

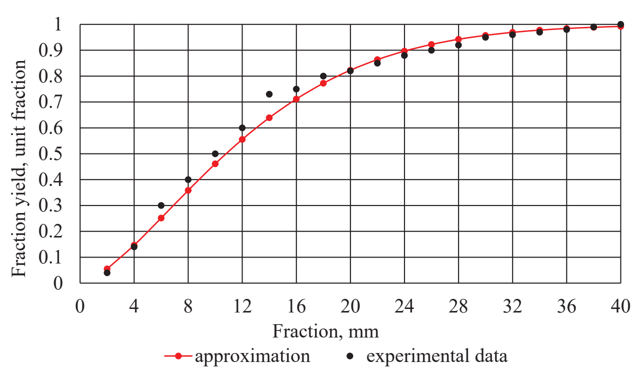

Fig. 1. Granulometric composition of the feed ore

Analysis of the granulometric composition of the feed ore enables a suggestion that the feed ore sizes are distributed according to Rosen-Rammler described by formula (28). To find the parameters from this formula, we apply the least-square method (LSM) [23]. To use the standard LSM program as the *Regression* function of the Excel software complex, we transform formula (28) to obtain a linear expression.

To do this, we apply the following actions:

$$1-F(d_0)=e^{-\left(\frac{d_0}{\Delta}\right)^n},\ \ln\left(1-F(d_0)\right)=-\left(\frac{d_0}{\Delta}\right)^n,$$

$$\ln\left(-\ln\left(1-F(d_0)\right)\right)=-n\ln\Delta+n\ln d_0. \quad (39)$$

Introducing the symbols into (39):

$$y=\ln\left(-\ln\left(1-F(d_0)\right)\right),\ a=-n\ln\Delta,\ x=n\ln d_0, \quad (40)$$

we obtain a linear function:

$$y=a+n\cdot x. \quad (41)$$

Table 1 presents the results of calculations by formulae (40).

Table 1

Input data to analyze the distribution function of ore sizes

No.	Fractions (d_0), mm	Empirical distribution function $F_n(d_0)$	x	y	Theoretical distribution function $F(d_0)$	$F_n(d_0)–F(d_0)\|$
1	2	0.04	0.69	–3.20	0.055	0.015
2	4	0.14	1.39	–1.89	0.147	0.007
3	6	0.3	1.79	–1.03	0.251	0.049
4	8	0.4	2.08	–0.67	0.359	0.041
5	10	0.5	2.30	–0.37	0.461	0.039
6	12	0.6	2.48	–0.09	0.555	0.045
7	14	0.73	2.64	0.27	0.639	0.091
8	16	0.75	2.77	0.33	0.711	0.039
9	18	0.8	2.89	0.48	0.772	0.028
10	20	0.82	3.00	0.54	0.823	0.003
11	22	0.85	3.09	0.64	0.864	0.014
12	24	0.88	3.18	0.75	0.897	0.017
13	26	0.9	3.26	0.83	0.922	0.022
14	28	0.92	3.33	0.93	0.942	0.022
15	30	0.95	3.40	1.10	0.958	0.008
16	32	0.96	3.47	1.17	0.969	0.009
17	34	0.97	3.53	1.25	0.978	0.008
18	36	0.98	3.58	1.36	0.984	0.004
19	38	0.99	3.64	1.53	0.989	0.001
20	40	1.00	3.66	1.54	0.992	0.008

Assessment of the parameters included in (41) provides the following results according to the MSL in Table 1.

$$a=-3.9,\ \ n=1.485. \quad (42)$$

According to (40) and considering (42), we find:

$$\Delta=e^{-\frac{a}{n}}=13.82. \quad (43)$$

Substituting (42) and (43) into formula (28), we obtain the analytical form of recording Rosen-Rammler's law of distributing the feed ore sizes:

$$F(d_0)=1-e^{-\left(\frac{d_0}{13.82}\right)^{1.485}}. \quad (44)$$

Table 1 and Fig. 1 provide the calculation results by formula (44).

It seems reasonable to check the hypothesis about the type of the distribution law chosen as theoretical by applying the goodness-of fit test, which is the simplest way to verify the hypothesis in our case [24]. The test is the maximum value of the absolute difference between the empirical distribution function and the relevant theoretical distribution function, in other words:

$$D=\max_{d_0}\left|F_n(d_0)-F(d_0)\right|. \quad (45)$$

[25] proves that for any type of continuous distribution function with an unlimited increase of the number of independent observations the probability of the inequation:

$$D\sqrt{n}\geq\lambda, \quad (46)$$

for tends to limit:

$$P(\lambda)=1-\sum_{k=-\infty}^{\infty}(-1)^k e^{-2k^2\lambda^2}. \quad (47)$$

For probability (47), there is a table presented in [25]. Using this table, we find the probability value (47) that corresponds to the fact that due to random reasons the maximum absolute deviation (45) will be no less than the actual observed one. If the probability value (47) is large enough, we can consider that the hypothesis about the distribution law is compatible with the experimental results.

According to the data presented in the last column of Table 1, the value (45) equals 0.091. Then, according to (46), we obtain:

$$\lambda_0=D\sqrt{n}=0.091\sqrt{20}=0.406.$$

Using the table in [24], we find:

$$P(\lambda_0)=0.995. \quad (48)$$

As the obtained probability (48) is large enough, we can conclude that the deviation between the empirical and theoretical distribution functions is insufficient. Thus, this difference can be explained by a random factor, i. e. the experimental data agree with the hypothesis about the ore fraction size being a random value with Rosen-Rammler's distribution.

According to (37), the differential function of this distribution is as follows:

$$f(d_0)=0.02985d_0^{0.485}e^{-(0.072d_0)^{1.485}}. \quad (49)$$

The diagram of the differential distribution function (49) is in Fig. 2. Considering (30), the mathematical expectation of the feed ore sizes is written as:

$$M[D_0]=\Delta\cdot\Gamma\left(1+\frac{1}{n}\right)=13.82\cdot\Gamma\left(1+\frac{1}{1.485}\right)=12.556. \quad (50)$$

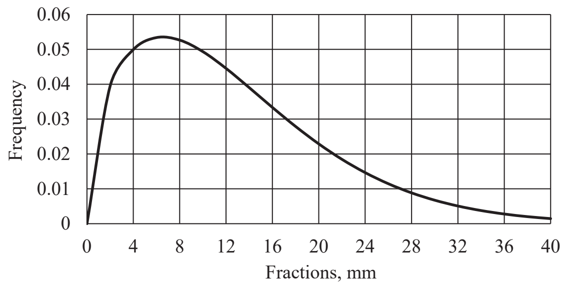

Fig. 2. Differential distribution function of the feed ore sizes

According to (31), dispersion of the feed ore sizes will be written as:

$$D[D_0]=13.82^2\left(\Gamma\left(1+\frac{2}{1.485}\right)-\Gamma\left(1+\frac{1}{1.485}\right)^2\right)=73.28. \quad (51)$$

According to (32) and (33), the integral and differential functions of the crush product sizes will be as follows:

$$G(d_1)=1-e^{-\left(\frac{0.072d_1}{\left(1-\frac{E_1}{k}\sqrt{d_1}\right)^2}\right)^{1.485}}, \quad (52)$$

$$g(d_1)=0.02985\left(\frac{d_1}{\left(1-\frac{E_1}{k}\sqrt{d_1}\right)^2}\right)^{0.485}\times$$

$$\times e^{-\left(\frac{0.072d_1}{\left(1-\frac{E_1}{k}\sqrt{d_1}\right)^2}\right)^{1.485}}\frac{1}{\left|1-\frac{E_1}{k}\sqrt{d_1}\right|^3}. \quad (53)$$

Formulae (52) and (53) result from the structural synthesis of the crushing model. To identify the parameter of the model k, one should solve the problem of finding functional extremum (34). The initial data on identifying the parameter of the model k is in Table 2. The parameter k is found by reducing the functional:

$$S(k)=\sum_{i=0}^{17}\left(G^*(d_{1i})-1-e^{-\left(\frac{0.072d_{1i}}{\left(1-\frac{E_1}{k}\sqrt{d_{1i}}\right)^2}\right)^{1.485}}\right)^2\to\min_k. \quad (54)$$

In functional (54), a single energy impact is accepted to correspond to the condition under which the data on the empirical distribution function of the crush product is collected. Application of the *root* function from Mathcad enables the optimal value of the parameter:

$$k^*=41.773. \quad (55)$$

Application of the goodness-of-fit test to the data of Table 2 results in the following:

$$\lambda=D\sqrt{n}=\max_{d_1}\left|G^*(d_1)-G(d_1)\right|\sqrt{n}=0.139\cdot\sqrt{18}=0.589.$$

According to the table from [24], the probability value equals:

$$P(0.589)=0.877. \quad (56)$$

As probability (56) is quite high, we can consider that the hypothesis about the distribution law complies with the experimental results.

Table 2

Data on identification of the model parameter

No.	Fractions (d_1), mm	Empirical distribution function G^* (d_1)	Results of identifying the model parameters G (d_1)	$\lvert G^*(d_1)-G(d_1)\rvert$
1	0	0	0	0
2	2	0.2	0.061	0.139
3	4	0.3	0.167	0.133
4	6	0.4	0.292	0.108
5	8	0.5	0.419	0.081
6	10	0.6	0.54	0.06
7	12	0.65	0.647	0.003
8	14	0.7	0.737	0.037
9	16	0.78	0.81	0.03
10	18	0.82	0.867	0.047
11	20	0.85	0.909	0.059
12	22	0.9	0.94	0.04
13	24	0.95	0.961	0.011
14	26	0.96	0.976	0.016
15	28	0.98	0.985	0.005
16	30	0.98	0.991	0.011
17	32	0.99	0.995	0.005
18	34	1	0.997	0.003

Fig. 3 presents the granulometric composition of the crush product calculated on the basis of the empirical distribution function and as a result of identifying the model parameter. Comparison of the diagrams of granulometric composition of the crush product from Fig. 3 reveals a good coincidence of the empirical distribution function and the results of identifying the model, being confirmed by the goodness-of-fit test.

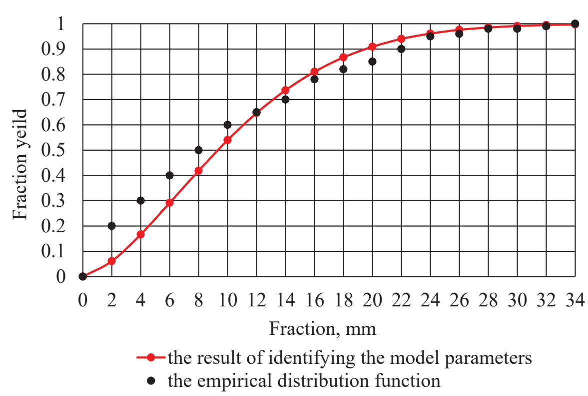

Fig. 3. Granulometric composition of the crush product

According to (52) and (53), the integral and differential distribution functions are as follows:

$$G(d_1)=1-e^{-\left(\frac{0.072d_1}{\left(1-0.0234\sqrt{d_1}\right)^2}\right)^{1.485}}, \quad (57)$$

$$g(d_1)=0.02985\left(\frac{d_1}{\left(1-0.0234\sqrt{d_1}\right)^2}\right)^{0.485}\times$$
$$\times e^{-\left(\frac{0.072d_1}{\left(1-0.0234\sqrt{d_1}\right)^2}\right)^{1.485}}\frac{1}{\left|1-0.0234\sqrt{d_1}\right|^3}. \quad (58)$$

The solution of the optimal stabilization problem involves the reduction of functional (45) in accordance with (58):

$$\int_0^{\left(\frac{41.773}{E_1}\right)^2}\left(x-d_1^*\right)^2\left(\frac{x}{\left(1-0.0234E_1\sqrt{x}\right)^2}\right)^{0.485}e^{-\left(\frac{0.072x}{\left(1-0.0234E_1\sqrt{x}\right)^2}\right)^{1.485}}\frac{1}{\left|1-0.0234E_1\sqrt{x}\right|^3}\mathrm{d}x\to\min_{E_1}, \quad (59)$$

where $d_1^*=4$ mm. Application of the root function from Mathcad enables the optimal value of the controlling impact:

$$E_1^*=10.495. \quad (60)$$

The quality of optimization determined by (38) makes:

$$\sigma^*=1.495. \quad (61)$$

The differential distribution function of the crush product sizes at the optimal controlling impact (60) is written as:

$$g(d_1)=0.02985\left(\frac{d_1}{\left(1-0.246\sqrt{d_1}\right)^2}\right)^{0.485}\times$$
$$\times e^{-\left(\frac{0.072d_1}{\left(1-0.246\sqrt{d_1}\right)^2}\right)^{1.485}}\frac{1}{\left|1-0.246\sqrt{d_1}\right|^3}. \quad (62)$$

Fig. 4 provides the diagrams of the differential functions of the crush product sizes of initial distribution set by formula (58) and the optimal distribution set by (62). Analysis of the diagrams in Fig. 4 reveals that with the optimal stabilization of the crush product sizes, the diagram of the differential distribution function of the crush product sizes «compresses» around the set stabilization value.

If numerical characteristics for initial distribution are equal:

$$M[D_1]=10.454,\quad D[D_1]=43.717, \quad (63)$$

the following is true for optimal distribution:

$$M^*[D_1]=3.28,\quad D^*[D_1]=1.717. \quad (64)$$

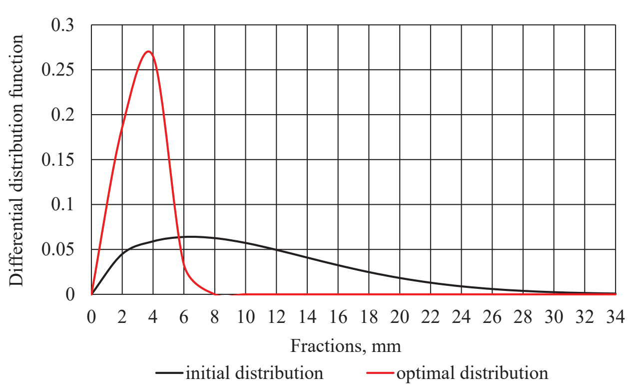

Fig. 4. Differential distribution functions of the crush product sizes

Comparison of (63) and (64) reveals that the stabilization aim is actually achieved. Dispersion that characterizes the average diameter of the crush product lumps, according to (63) and (64), reduces by a factor of 25.46.

7. Discussion of results of mathematical simulation of roller-press ore crushing

The synthesized mathematical model of ore crushing in the roller-press enables considering stochastic properties of the feed material set by the differential distribution function (15). Presentation of the simulation result by means of the differential distribution function (16) allows formulating the problem of optimal stabilization as for the set average diameter of the crush product lump as an extreme problem in the stochastic form (26). The statistics-based results of simulation of oxidized iron ores crushing presented in Tables 1, 2 confirm the relevancy of the developments to be implemented.

Meanwhile, there are some complications associated with the assessment of the stochasticity of the ore flow feeding the roller-press as it can be dynamic. The Bond's crushing law applied to describing roller-press crushing causes some constraints. Consideration of all these aspects requires involving particular conditions of ore mining and crushing.

The drawback of the suggested method is the absence of any chance to optimize roller-press functioning in engineering conditions. That is why, further research should include the obtained results into the automated control system of a concentration plant as its real-time functioning subsystem.

8. Conclusions

1. The mathematical model of roller-press crushing is developed, its peculiarity being the description of ore disintegration by means of Bond's crushing law. Unlike the determined approaches, it considers the stochasticity of the ore flow feeding the roller-press to describe sizes of the crush product lumps as a random value. On the basis of the crushing results, it reveals how to identify the model parameters. The synthesized model is noted for the possibility to find the probabilistic law of distributing sizes of the crush product lump in the roller-press. This distribution law enables both assessing numerical characteristics of the product lump size and formulating probabilistically the problem of optimal stabilization of the crushing process. A simulation that considers the stochasticity of the feed ore flow distributed according to Rosen-Rammler's law enables analytical formulae describing the crushing process in the roller-press followed up by formulating the optimal stabilization problem.

2. The optimal stabilization problem of roller-press crushing formulated probabilistically, i. e. considering stochastic properties of the feed materials, is solved. Statistics-based simulation of oxidized iron ores crushing both reveals the feasibility of the developed approach and confirms optimization results by the goodness-of-fit test statistically. Thus, optimal stabilization allows achieving the control aim. The crushed ore size makes 3.28 mm against the expected 4 mm, while dispersion characterizing the average diameter of the crush product lumps reduces by a factor of 25.46.

Reference

1. Oliynyk, T. A., Skliar, L. V., Oliynyk, M. O. (2015). Doslidzhennia umov selektyvnoho ruinuvannia mineralnykh kompleksiv pry zbahachenni hematyt-ilmenitovykh rud. Zbahachennia korysnykh kopalyn, 60 (101), 57–67.
2. Linch, A. (1981). Tsikly drobleniya i izmel'cheniya. Modelirovanie, optimizatsiya, proektirovanie i upravlenie. Moscow: Nedra, 343.
3. Elnikova, S. P., Gazaleeva, G. I. (2019). Reduction energy prediction for layer-type cone crushers. Obogashchenie Rud, 5, 3–8. doi: https://doi.org/10.17580/or.2019.05.01
4. Vasil'ev, P. (2012). Chislennoe reshenie uravneniya kinetiki dezintegratsii i raskrytiya mineralov polikristallicheskih chastits. Nauchnye vedomosti Belgorodskogo gosudarstvennogo universiteta. Seriya: Ekonomika. Informatika, 7 (126), 92–101.
5. Saramak, D. (2013). Mathematical models of particle size distribution in simulation analysis of highpressure grinding roll operations. Physicochemical Problems of Mineral Processing, 49 (1), 121–131. doi: http://doi.org/10.5277/ppmp130112
6. Anticoi, H., Guasch, E., Ahmad Hamid, S., Oliva, J., Alfonso, P., Bascompta, M. et. al. (2018). An Improved High-Pressure Roll Crusher Model for Tungsten and Tantalum Ores. Minerals, 8 (11), 483. doi: https://doi.org/10.3390/min8110483
7. Anticoi, H., Guasch, E., Oliva, J., Alfonso, P., Bascompta, M., Sanmiquel, L. (2019). High-pressure grinding rolls: model validation and function parameters dependency on process conditions. Journal of Materials Research and Technology, 8 (6), 5476–5489. doi: https://doi.org/10.1016/j.jmrt.2019.09.016
8. Djokoto, S. S., Karimi, H. R. (2011). Modeling and simulation of a High Pressure Roller Crusher for silicon carbide production. 11th International Conference on Electrical Power Quality and Utilisation. doi: https://doi.org/10.1109/epqu.2011.6128963
9. Rashidi, S., Rajamani, R. K., Fuerstenau, D. W. (2017). A Review of the Modeling of High Pressure Grinding Rolls. KONA Powder and Particle Journal, 34, 125–140. doi: https://doi.org/10.14356/kona.2017017
10. Fedotov, P. K., Pykhalov, A. A. (2014). Numerical simulation of boundary conditions and load in analysis of a breed's stress-strain when splitting it between rollers in roller press. Sovremennye tehnologii. Sistemniy analiz. Modelirovanie, 1 (37), 27–32.
11. Fedotov, P. K. (2014). Modeling fracture of ore particles in a layer under pressure. Journal of Mining Science, 50 (4), 674–679. doi: https://doi.org/10.1134/s1062739114040073
12. Fedotov, P. (2013). Main reason of lower energy consumption by ore fragmentation in press-rollers. Gorniy informatsionno-analiticheskiy byulleten', 3, 309–314.
13. Hopunov, E. (2011). Rol' struktury i prochnostnyh harakteristik mineralov v razrushenii i raskrytii rud. Obogashchenie rud, 1, 25–31.
14. Mulyavko, V., Oleynik, T., Oleynik, M., Mikhno, S., Lyashenko, V. (2014). Innovation technologies and machinery for separation of feebly magnetic ores. Obogashchenie Rud, 2 (350), 43–49.
15. Morkun, V., Gubin, G., Oliinyk, T., Lotous, V., Ravinskaia, V., Tron, V. et. al. (2017). High-energy ultrasound to improve the quality of purifying the particles of iron ore in the process of its enrichment. Eastern-European Journal of Enterprise Technologies, 6 (12 (90)), 41–51. doi: https://doi.org/10.15587/1729-4061.2017.118448
16. Smyrnov, V. O., Biletskyi, V. S. (2012). Pidhotovchi protsesy zbahachennia korysnykh kopalyn. Donetsk: Skhidnyi vydavnychyi dim, Donetske viddilennia NTSh, 284.
17. Zaytsev, V., Polyanin, A. (2001). Spravochnik po obyknovennym differentsial'nym uravneniyam. Moscow: Fizmatlit, 576.
18. Izmailov, A., Solodov, M. (2008). Chislennye metody optimizatsii. Moscow: Fizmatlit, 320.
19. Alekseev, V., Tihomirov, V., Fomin, S. (2005). Optimal'noe upravlenie. Moscow: Fizmatlit, 384.
20. Lavrent'ev, M., Shabat, B. (1973). Problemy gidrodinamiki i ih matematicheskie modeli. Moscow: Fizmatlit, 416.
21. Olver, F. (1978). Vvedenie v asimptoticheskie metody i spetsial'nye funktsii. Moscow: Nauka, 375.
22. Oliinyk, T., Nikolaienko, P. (2019). Vykorystannia matematychnoho modeliuvannia dlia vyznachennia hranulometrychnoho skladu produktu droblennia rudy v shari pid tyskom. Zbahachennia korysnykh kopalyn, 72 (113), 44–47.
23. Linnik, Yu. (1962). Metod naimen'shih kvadratov i osnovy matematiko-statisticheskoy teorii obrabotki nablyudeniy. Moscow, 354.
24. Id'e, V., Drayard, D., Dzheyms, F., Rus, M., Sadule, B. (1976). Statisticheskie metody v eksperimental'noy fizike. Moscow: Atomizdat, 335.
25. Dimitrova, D. S., Kaishev, V. K., Tan, S. (2017). Computing the Kolmogorov-Smirnov Distribution when the Underlying CDF is Purely Discrete, Mixed or Continuous. Available at: https://openaccess.city.ac.uk/id/eprint/18541/8/Dimitrova%252C%20Kaishev%252C%20Tan%20%25282017%2529%20KSr1.pdf