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# Mathematical modeling of shock wave interaction with wagon

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**Abstract.** In case of some accidents on railways there may be situations when the shock wave appears and interacts with different objects on the railway (wagons, cargo, buildings etc.). In these cases it is necessary to predict the possible effect of shock wave diffraction on the different objects. Study of these problems on the basis of physical experiments (laboratory experiment or field experiment) demand expensive and unique experimental facility. In some cases physical experiment can't be set. That is why mathematical simulation plays the important role in solving problem connected with shock wave propagation. For practice it is necessary to have predictive quick computing mathematical models which allow to perform numerical experiment on the basis of non-powerful computers. Now, in Ukraine, there is a real deficit of mathematical models which allow to compute quickly shock wave interaction with different objects. The aim of this work was development of quick computing numerical model to simulate shock wave propagation and its interaction with the wagons. The model is based on the numerical integration of Euler equations which are written in integral form. To solve modeling equations difference scheme of splitting was used. Results of numerical modeling are presented.

## 1. Introduction

Explosion of railway tanks may cause shock wave appearance. Also, use of explosives at railway stations results in shock wave appearance and wave interaction with objects. That is why, shock wave interaction with different objects on the railway stations, shock wave interaction with wagons are very important problems in the field of mechanics [1-5]. Worthy of note, that mathematical modeling is the main tool to solve this problem because of physical experiment can't be used for every day study this complicated problem. Study diffraction processes of shock wave propagation needs very expensive equipment, special experimental fields and sometimes it can't be organized for the problems which are under interest.

Two approaches are used in the field of mathematical modeling of shock wave interaction with different objects. The first way is based on the numerical integration of Euler equations. Modeling



shock wave interaction on the basis Euler equations does not take into account viscous effects. Computation time when this approach is used is small. That is very convenient to perform series of numerical experiments during one working day. The second way is based on the numerical integration Navier-Stokes equations. This approach needs much computational time. On practice, different commercial codes are used to model shock wave interaction with different objects.

The goal of this work is development numerical model on the basis of Euler equations for modeling shock wave interaction with objects on the railway transport.

## 2. Mathematical model

We consider shock wave propagation on the railway station. We model this shock wave interaction with the cargo which is situated on the railway platform (figure 1). Near this railway platform there is another track with cargo train. It means that, in common, we model the situation of shock wave diffraction on two objects.



**Figure 1.** Sketch of computational region: 1 – shock wave front ([http://www.fpk.ru/perevozka\\_tehniki.html](http://www.fpk.ru/perevozka_tehniki.html)).

To simulate the shock wave diffraction on two objects we used Euler equations which were written in the form of integral laws of conservation. These equations were written for the non-orthogonal difference cell [6]:

$$\frac{\partial}{\partial t} \oint_{\Delta G} \rho d\tau + \oint_{\Delta \Gamma} \rho V_n ds = 0 \quad (1)$$

$$\frac{\partial}{\partial t} \oint_{\Delta G} \rho \vec{V} d\tau + \oint_{\Delta \Gamma} \rho \vec{V} V_n ds + \oint_{\Delta \Gamma} P \vec{n} ds = 0 \quad (2)$$

$$\frac{\partial}{\partial t} \oint_{\Delta G} \rho E d\tau + \oint_{\Delta \Gamma} \rho I V_n ds = 0 \quad (3)$$

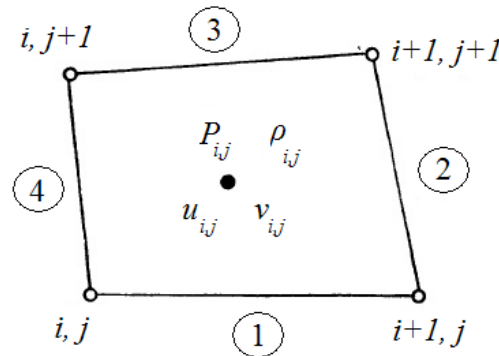
where  $\Delta G$  is square of the computational cell;  $\Delta \Gamma$  is the boundary of this computational cell;  $P$  is pressure;  $u, v$  are components of air velocity vector;  $\rho$  is density;  $E = \frac{u^2}{2} + \frac{v^2}{2} + \frac{P}{\rho(1-\gamma)}$  is internal

energy;  $I = E + \frac{P}{\rho}$  is enthalpy.

Initial and boundary conditions are considered in [6, 7]. Gas parameters behind the shock wave front are determined using Rankin-Gugonoio relationships [7].

### 3. Numerical model

The computational region is covered by non-orthogonal difference grid. Sketch of non-orthogonal computational cell is shown in figure 2 [7].



**Figure 2.** Sketch of non-orthogonal computation cell.

The numerical procedure to solve equations (1)-(3) was as following [7].

At first step we approximated the modeling equations as follows:

$$\frac{\rho_{ij}^{n+1} - \rho_{ij}^n}{\Delta t} \Delta G_{ij} + \sum_{k=1}^4 (\rho_k^+ W_k^+ + \rho_k^- W_k^-)^{n+1} \Delta s_k = 0, \tag{4}$$

$$\begin{aligned} \frac{\rho \vec{V}_{ij}^{n+1} - \rho \vec{V}_{ij}^n}{\Delta t} \Delta G_{ij} + \sum_{k=1}^4 ((\rho \vec{V})_k^+ W_k^+ + (\rho \vec{V})_k^- W_k^-)^{n+1} \Delta s_k + \\ + \sum_{k=1}^4 P_k^{n+1} \vec{N}_k \Delta s_k = 0, \end{aligned} \tag{5}$$

$$\frac{\rho E_{ij}^{n+1} - \rho E_{ij}^n}{\Delta t} \Delta G_{ij} + \sum_{k=1}^4 ((\rho I)_k^+ W_k^+ + (\rho I)_k^- W_k^-) \Delta s_k = 0, \tag{6}$$

where  $\rho_k^+ W_k^+ = \rho_{ij}^{n+1} \frac{1}{2} (V_{N,k}^{n+1} + |V_{N,k}^{n+1}|)$ ,

$$\rho_k^- W_k^- = \begin{cases} \rho_{i,j-1}^{n+1} & \text{for } k=1 \\ \rho_{i+1,j}^{n+1} & \text{for } k=2 \\ \rho_{i,j+1}^{n+1} & \text{for } k=3 \\ \rho_{i-1,j}^{n+1} & \text{for } k=4 \end{cases} \frac{1}{2} (V_{N,k}^{n+1} - |V_{N,k}^{n+1}|), \tag{7}$$

$$V_{N,k}^{n+1} = \frac{1}{2}(\vec{V}_{ij} \cdot \vec{N}_k)^{n+1} + \frac{1}{2} \left( \begin{array}{l} \vec{V}_{i,j-1} \text{ for } k=1 \\ \vec{V}_{i+1,j} \text{ for } k=2 \\ \vec{V}_{i,j+1} \text{ for } k=3 \\ \vec{V}_{i-1,j} \text{ for } k=4 \end{array} \right) \cdot \vec{N}_k, \quad (8)$$

where  $\vec{N}_k$  – is external normal to the « $k$ » part of the cell boundary.

The pressure on the cell boundary is determined as:

$$P_k^{n+1} = \frac{1}{2} P_{ij}^{n+1} + \frac{1}{2} \left( \begin{array}{l} P_{i,j-1} \text{ for } k=1 \\ P_{i+1,j} \text{ for } k=2 \\ P_{i,j+1} \text{ for } k=3 \\ P_{i-1,j} \text{ for } k=4 \end{array} \right). \quad (9)$$

If we use the following designations:

$$T_k^+ \rho_{ij}^{n+1} = \frac{1}{\Delta G_{ij}} \rho_k^+ W_k^+ \Delta S_k, \quad (10)$$

$$T_k^- \rho_{ij}^{n+1} = \frac{1}{\Delta G_{ij}} \rho_k^- W_k^- \Delta S_k, \quad (11)$$

$$L_k P_{ij}^{n+1} = P_k^{n+1} \Delta S_k \frac{1}{\Delta G_{ij}}, \quad (12)$$

Using expression (7)-(12) we can write the difference equations as:

$$(E + \Delta t \sum_{k=1}^4 (T_k^+ + T_k^-)) \rho_{ij}^{n+1} = E \rho_{ij}^n, \quad (13)$$

$$(E + \Delta t \sum_{k=1}^4 (T_k^+ + T_k^-)) \rho \vec{V}_{ij}^{n+1} + \Delta t \sum_{k=1}^4 L_k P_{ij}^{n+1} = E \rho \vec{V}_{ij}^n, \quad (14)$$

$$E \rho E_{ij}^{n+1} + \Delta t \sum_{k=1}^4 (T_k^+ + T_k^-) \rho I_{ij}^{n+1} = E \rho E_{ij}^n. \quad (15)$$

At the second step we write the difference scheme of splitting:

$$\left( E + \frac{\Delta t}{2} (T_4^- + T_2^+ + T_1^- + T_3^+) \right) \rho_{ij}^{n+\frac{1}{4}} = E \rho_{ij}^n, \quad (16)$$

$$\left( E + \frac{\Delta t}{2} (T_2^- + T_4^+ + T_3^- + T_1^+) \right) \rho_{ij}^{n+\frac{1}{2}} = E \rho_{ij}^{n+\frac{1}{4}}, \quad (17)$$

$$\left( E + \frac{\Delta t}{2} (T_4^- + T_2^+ + T_3^- + T_1^+) \right) \rho_{ij}^{n+\frac{3}{4}} = E \rho_{ij}^{n+\frac{1}{2}}, \quad (18)$$

$$\left( E + \frac{\Delta t}{2} (T_2^- + T_4^+ + T_1^- + T_3^+) \right) \rho_{ij}^{n+1} = E \rho_{ij}^{n+\frac{3}{4}}, \quad (19)$$

$$\left(E + \frac{\Delta t}{2} T^{++}\right) \rho \vec{V}_{ij}^{n+\frac{1}{4}} + \frac{\Delta t}{4} \sum_{k=1}^4 L_k P_{ij}^{n+\frac{1}{4}} \vec{N}_k = E \rho \vec{V}_{ij}^n a, \quad (20)$$

$$\left(E + \frac{\Delta t}{2} T^{--}\right) \rho \vec{V}_{ij}^{n+\frac{1}{2}} + \frac{\Delta t}{4} \sum_{k=1}^4 L_k P_{ij}^{n+\frac{1}{2}} \vec{N}_k = E \rho \vec{V}_{ij}^{n+\frac{1}{4}}, \quad (21)$$

$$\left(E + \frac{\Delta t}{2} T^{+-}\right) \rho \vec{V}_{ij}^{n+\frac{3}{4}} + \frac{\Delta t}{4} \sum_{k=1}^4 L_k P_{ij}^{n+\frac{3}{4}} \vec{N}_k = E \rho \vec{V}_{ij}^{n+\frac{1}{2}}, \quad (22)$$

$$\left(E + \frac{\Delta t}{2} T^{-+}\right) \rho \vec{V}_{ij}^{n+1} + \frac{\Delta t}{4} \sum_{k=1}^4 L_k P_{ij}^{n+1} \vec{N}_k = E \rho \vec{V}_{ij}^{n+\frac{3}{4}}, \quad (23)$$

$$E \rho E_{ij}^{n+\frac{1}{4}} + \frac{\Delta t}{2} T^{++} \rho I_{ij}^{n+\frac{1}{4}} = E \rho E_{ij}^n, \quad (24)$$

$$E \rho E_{ij}^{n+\frac{1}{2}} + \frac{\Delta t}{2} T^{--} \rho I_{ij}^{n+\frac{1}{2}} = E \rho E_{ij}^{n+\frac{1}{4}}, \quad (25)$$

$$E \rho E_{ij}^{n+\frac{3}{4}} + \frac{\Delta t}{2} T^{+-} \rho I_{ij}^{n+\frac{3}{4}} = E \rho E_{ij}^{n+\frac{1}{2}}, \quad (26)$$

$$E \rho E_{ij}^{n+1} + \frac{\Delta t}{2} T^{-+} \rho I_{ij}^{n+1} = E \rho E_{ij}^{n+\frac{3}{4}}, \quad (27)$$

In expressions (24)-(27) we use the following designations:

$$T^{++} = T_4^- + T_2^+ + T_1^- + T_3^+, \quad (28)$$

$$T^{--} = T_2^- + T_4^+ + T_3^- + T_1^+, \quad (29)$$

$$T^{+-} = T_4^- + T_2^+ + T_3^- + T_1^+, \quad (30)$$

$$T^{-+} = T_2^- + T_4^+ + T_1^- + T_3^+. \quad (31)$$

If we eliminate the fractional steps we obtain the following scheme [7]:

$$\left(E + \frac{\Delta t}{2} T^{+-}\right) \left(E + \frac{\Delta t}{2} T^{-+}\right) \rho_{ij}^{n+1} = E \rho_{ij}^{n+\frac{1}{2}}, \quad (32)$$

$$\left(E + \frac{\Delta t}{2} T^{--}\right) \left(E + \frac{\Delta t}{2} T^{+-}\right) \left(E + \frac{\Delta t}{2} T^{-+}\right) \rho_{ij}^{n+1} = E \rho_{ij}^{n+\frac{1}{4}}, \quad (33)$$

$$\left(E + \frac{\Delta t}{2} T^{++}\right) \left(E + \frac{\Delta t}{2} T^{--}\right) \left(E + \frac{\Delta t}{2} T^{+-}\right) \left(E + \frac{\Delta t}{2} T^{-+}\right) \rho_{ij}^{n+1} = E \rho_{ij}^n. \quad (34)$$

As this numerical scheme has the first order of accuracy the shock wave front is «smeared» on some computational cells.

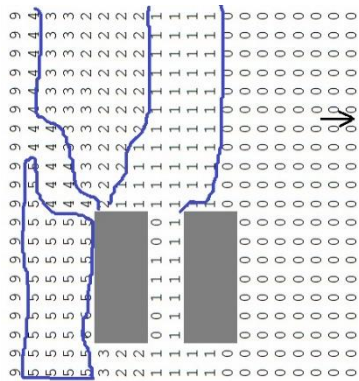
FORTRAN was used to develop code on the basis of described numerical model. Developed code includes the following SUBPROGRAMs:

1. «DENS» – subprogram to compute density field in computational domain.
2. «VEL» – subprogram to compute air velocity components field in computational domain.
3. «PRES» – subprogram to compute pressure field in computational domain.

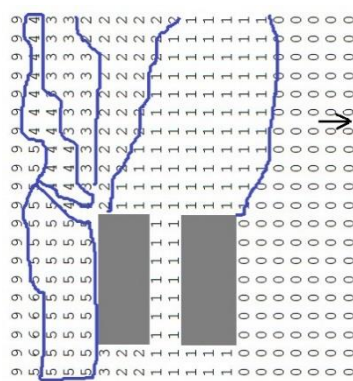
4. «P5GF» – subprogram to illustrate parameter distribution in computational domain.  
Coordination of all subprograms is performed by main program.

#### 4. Results

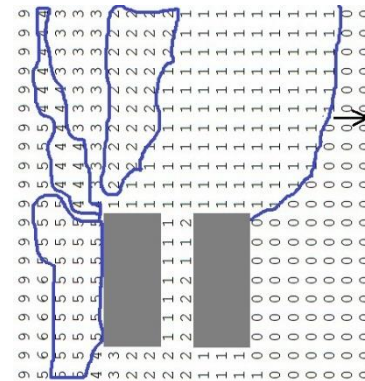
Numerical experiment was performed for scenario when the pressure behind the shock wave front was 15 atm. Dimensionless pressure field for different time of shock wave diffraction on two objects is shown in figures 3-5. Time is also dimensionless.



**Figure 3.** Pressure field for time  $t=0.43$ .



**Figure 4.** Pressure field for time  $t=0.56$ .



**Figure 5.** Pressure field for time  $t=0.69$ .

Each «number» in figures 3-5 shows the range of pressure value. For example, number «1» means that in this region pressure is in the range of about 10% of the maximum pressure at this time. As we see from the figures 3-5, the pressure field near the objects has the complicated form which is caused by shock wave interaction with objects. We see that pressure rises near the first obstacle caused by gas flow breaking on the surface of this object. Worthy of note that computational time was 3s.

#### 5. Conclusions

Briefly, numerical model was developed to simulate shock wave interaction with objects on the railway station. To model shock wave diffraction Euler equations are used. The model can be used for preliminary evaluation of shock wave impact on different objects. Further, we shall develop model to solve conjugate problem: «reaction of object + load from shock wave».

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