The use of new structural solutions of retaining walls to ensure the stable operation of the "base – engineering structure" system

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Abstract. During the design and operation practice, it is necessary to assess the possibility of deformation of structures located on foundations which are able to precipitate and subside. Only with reliable and accurate determination of the stress-strain state of structures of structures together with soil masses during modeling, it is possible to apply the finite element method in practice. The calculations using the PLAXIS and LIRA programs made it easier to analyze the stress-strain state of the soil mass and the stability of retaining walls: I option is corner retaining wall; II option is retaining wall with a structural surface. With the same soil base (layer geometry and physicomechanical properties), loads and boundary load conditions, it is obvious that for the II option the entire mass of soil is included in the work and the stresses are uniformly distributed over the front and foundation plates (over common stresses), uniform structural deformations are observed and soil base, which, in turn, ensures the stability of the retaining wall (according to the general picture of movements) The validity of the theoretical forecast of the behavior of engineering structures interacting with an unevenly deformed base cannot be obtained on the basis of the regulatory framework. This gap can be filled in when modeling the "base – engineering structure" system using modern calculation programs using the finite element method.

1 The problem and its relationship with scientific and practical tasks

Retaining walls are now widely used not only in civil and industrial construction, but also in urban planning for complex landscapes.

There are situations of construction of objects in cramped conditions, in adverse territories with active acting deformation influences, which makes the use of existing types of retaining wall structures more complicated. An important factor in solving the emerging problems is the high level of expertise on engineering and geological conditions in large industrial cities, especially in regional centers. Modern research is mainly focused on clarifying the geotechnical nature [1].

As earlier experience shows, the development of hazardous processes in urban areas continues. First of all, these are the processes of flooding in the territories, the development of landslide deformations, subsidence of the surface above mine workings, subsidence of loess soils, etc. These processes are associated with a decrease in serviceability or deformation and the destruction of buildings and structures.

According to expert estimates, 90% of the territory of Ukraine is characterized by complex engineering and

geological conditions, worsening due to the influence of natural and technogenic factors.

As for the Kryvyi Rih city, the technogenic load on the geological environment is several orders of magnitude higher than for other adjacent cities. It should be especially noted that iron ore has been mined for about 200 years, up to 47% of the built-up territory is located in the developed space. In modern practice, the construction of facilities in cramped conditions, in areas subject to adverse physical and geological processes, complicates the use of existing types of retaining wall structures. Extremely unfavorable combinations of underworking with subsidence of soils, with flooding, etc. are encountered, therefore, in order to protect the settlements, industrial facilities, utilities and transport communications with a certain level of safety, it is necessary to use engineering structures of a special type that would perceive the influence of variables of engineering -geological, natural and technogenic factors.

Building on a deformable foundation is very widespread. In recent years, as the territories that are most favourable for development have been exhausted, construction in territories with difficult conditions, which are characterized by significant uneven deformability of the base, has become increasingly widespread [2, 3].

Engineering methods of calculation, which are traditionally used in design, cannot answer all questions

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of operational practice. Widespread in many countries are programs based on the finite element method (FEM). These programs have a fairly wide scope and according to them it is possible to determine the stress-strain state, including structures, in the "base – engineering structure" system [4-7].

2 Analysis of research and publications

Complicated engineering and geological conditions - this is a geological environment that includes specific soils, processes, hazardous natural technogenic or geomorphological geological conditions, and hydrogeological factors of interaction with buildings and structures, belong to the II and III category of complexity of engineering and geological conditions [8, 9]. Many scientists have devoted their work to determining the lateral pressure of soil on retaining walls, taking into account their joint work with the soil mass.

So, V. Raiuk [10, 11] investigated the nature and magnitude of the lateral pressure on the vertical face of the retaining wall, taking into account its deformation and displacement, using the model of linearly deformable half-space, but he did not consider the joint work of the vertical wall and the foundation as a single system that interacts with soil. K. Chernyshova [12] investigated the effect of the flexibility of a vertical wall on the lateral pressure of the soil.

Y. Luchkovskyi [13], using the superposition method, provides a solution for determining the lateral pressure of soil on retaining walls from narrow loads and concentrated force. He draws attention to the attenuation with a deepening of lateral soil pressure from the load. However, these authors, when determining the lateral pressure, did not take into account the joint work of the vertical wall and the foundation as a single system that interacts with the base.

Y. Symvulydy [14] calculates a flexible retaining wall taking into account the interaction of all its elements with the base, but introduces a linearly deformable half-plane as a model of the natural base, which is not entirely correct. In addition, this method does not allow taking into account the influence of the loading on the stress-strain state of the system.

A significant amount of research has been devoted to the static calculation of retaining walls [15, 16]. The number of works on the dynamics of these structures, especially taking into account the elastic properties of their materials, is much smaller [17, 18]. Usually, dynamic calculations are performed if pulsed, vibrational or moving loads act on the structure [19].

3 Formulation of the problem

When constructing retaining walls in complex engineering and geological conditions, it is necessary to achieve:

- increasing the stability and strength of retaining walls;

reducing the cost of used building materials;

- decreasing in the volume of earthwork; reduction of strain unevenness;

- reduction of construction time;

- improving the conditions of filling and compaction of the filling;

- increasing operational reliability, quality of work and increase the service life of retaining walls.

Thus, to assess the stress-strain state of retaining walls, it is necessary to take into account the joint work of the entire wall with soil and the use of more reasonable soil models in the region of its vertical and horizontal elements.

The aim of the research is to study the stress-strain state of the base and retaining wall with a structural surface (PSSP) taking into account their contact interaction to increase the stable operation of the "base – engineering structure" system.

4 Statement of material and results

The most common engineering structures in urban design today are retaining walls that are used for fencing:

- slopes of embankments and excavations within the area and access railways and roads in case of inability of carrying out slopes with the necessary inclination;

- pits during the construction process when it is impossible to perform slopes with the necessary inclination;

- special structures - ramps, bulk materials warehouses, bunker overpasses and ore yards of metallurgical plants;

- individual overpriced or underpriced sections of technology located within and outside buildings according to technology conditions.

Since ancient times, massive retaining walls of stone blocks and slabs have been used to support slopes, excavations, embankments and natural slopes. Technical progress and the laws of economic efficiency draw attention to the design features of the retaining wall and the characteristics of the interaction of the foundation with it. Important issues are optimizing the design of retaining walls, taking into account the following characteristics:

- the maximum inclusion in the work of soil filling;

- decreasing the values of the active soil pressure;

- increasing resistance of the displacement along the sole of the retaining wall;

– minimization of material consumption.

An important factor in solving the tasks is a high level of knowledge of engineering and geological conditions in large industrial cities and regional centers. Modern research is aimed at clarifying mainly geotechnical nature. The experience of recent years shows that the development of hazardous processes in urban areas continues. First of all, these are the processes of flooding of territories, the development of shear deformations, subsidence of the surface above mine workings, subsidence of loess soils, etc. These processes are associated with a decrease in serviceability or deformation, destruction of buildings and structures [2].

It is not always possible to use well-known technical solutions in the undermined territories and subsiding soils with complex deformations of the base, since they are not suitable for these working conditions. The existing structures of retaining walls are not designed for additional efforts from horizontal displacement or vertical movement of the soil, which causes stress concentration in the lower part of the front plate and, of course, will lead to the destruction of the structure.

Therefore, there was a need for the use of new designs that would take into account these shortcomings and increase the reliability of operation of unsuitable territories.

A retaining wall is proposed, which can be used to stabilize unstable slopes and gradients, as well as in undermined areas with horizontal and vertical movement of soil (Fig. 1–4) [20].

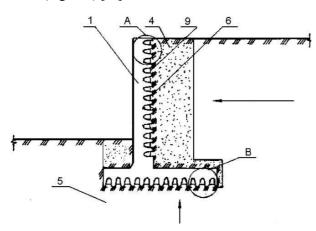


Fig. 1. Structural retaining wall.

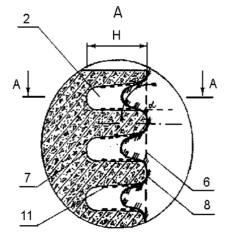


Fig. 2. Node A.

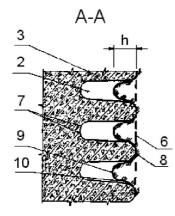


Fig. 3. Section A-A.

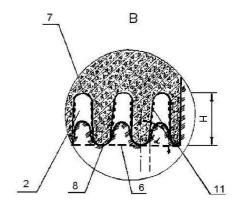


Fig. 4. Node B.

The new design of the retaining wall with a structural surface can be used to protect the territory from collapse. It represents an improvement of the monolithic retaining wall of the corner type by forming contact surfaces with cavities on the vertical and foundation elements from the ground, which allows to reduce the peaks of contact stresses on the surface of the vertical element of the retaining wall due to the uniform redistribution of pressure in the approaching soil, to increase the shear resistance along the sole retaining wall, in addition, in the case of additional uneven deformations of the base acting on the foundation the element, to improve its operation, which allows a retaining wall to perceive and evenly redistribute the efforts of complex loadings.

The monolithic retaining wall of the corner type includes vertical and foundation elements, and on the surface of the vertical and foundation elements, alternating support parts and cavities are placed on the contact side, while the cavities are made in the form of truncated pyramids of the same size and directed by a smaller base into the vertical element, while the volume cavities is determined by the formula:

$$V = \frac{2 \cdot \omega_{\phi} \cdot N \cdot H \cdot (K \cdot (\delta + h) - h)}{n \cdot R \cdot (H - \delta - h) \cdot [1 + (\cos \alpha + f \sin \alpha)]},$$
 (1)

where: V is the volume of cavities; N is the external normal load relative to the envelope surfaces of the supporting parts, which acts horizontally; R is the ultimate bearing capacity of the soil at which the static equilibrium of the retaining wall is established and maintained; H is total depth of the cavities; h – is the depth of penetration of the soil into the cavity; δ – is the absolute value of the forced displacement of the soil at the considered point; S is the actual contact area of the supporting prismatic sections; K – is the safety factor for the contact area, which takes into account possible changes in the estimated power loads; f – is the coefficient of friction between the soil and the supporting prismatic sections; ω_{d} – is the generalized coefficient for taking into account the shape of the volume of prismatic sections and their projection area along the envelope surfaces of the supporting parts; n - is the coefficient of conversion of contact pressures into their projection on the normal axis to the envelope surface of the supporting parts; α – is the angle formed by the envelope of the plane of the supporting parts and the tangent plane, which is drawn to the surface of the prismatic sections at a height *h*.

For a smooth perception of deforming actions from moving soil to a vertical element and deforming actions from vertical movement of soil to a foundation element, lateral faces of the planes are rounded. To reduce the effective friction forces on the vertical element, lateral faces of the planes are coated with an antifriction layer. To compact the backfill soil and to prevent the penetration of soil into the cavity, a sheet of resilient material is placed on the contour surfaces of the supporting parts.

The proposed retaining wall 1 consists of a vertical element, which has cavities 2 and supporting parts 3, in the direction of action of the shifting soil, and a foundation element, which has cavities 2 and supporting parts 3, in the direction of action of the vertically moving soil.

The backfill soil 4 and the base soil 5 are directed into cavities 2, which have the shape of truncated pyramids, with bases 6 and side faces 7. Side faces 7 and cavities 2 constitute the actual side surfaces of the support parts 3.

Facets 7 form ribs 8 in the plane of the bases 6, which make up the envelope surface of the supporting parts 3 (Fig. 5-8).

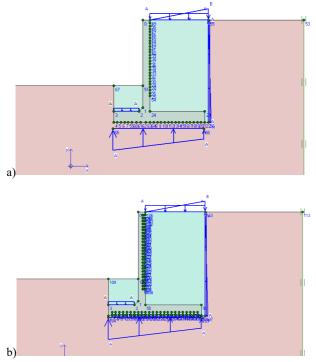
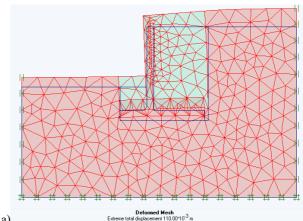


Fig. 5. Design schemes: a) I option; b) II option.

On the enveloped vertical surfaces of the supporting parts, between the soil and the cavities, depending on the type of soil conditions and the nature of the deformations, a sheet of resilient material 9 can be placed.

The soil directed into the cavity is in contact with the side surfaces of the supporting parts 3 by the supporting prismatic sections 10 of the side faces 7 of the cavities 2. An antifriction layer 11 can be made on the surface of the supporting parts, which is placed on the surface of the retaining wall.

With the development of the deforming load, that is, with vertical and horizontal movement of the soil relative to the retaining wall 1, after its installation, the soil penetrates into the cavity.





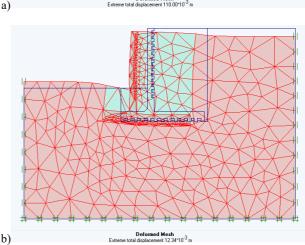


Fig. 6. Deformation of the mesh of finite elements: a) I option; b) II option.

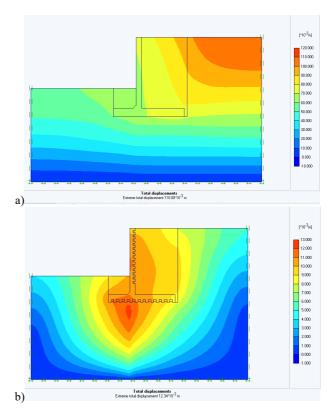


Fig. 7. Complete movements: a) I option; b) II option.

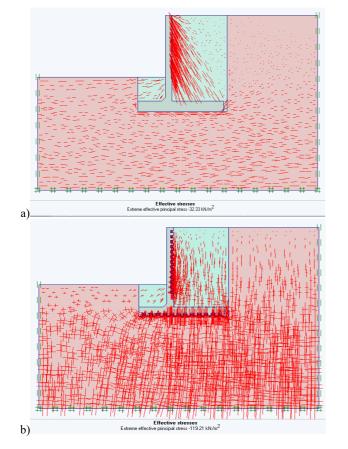


Fig. 8. Effective stress σ_{ef} : a) I option; b) II option.

Regarding the vertically built enveloping planes of the supporting parts 3, the vertical and foundation elements of the retaining wall 1, the acting loads in ordinary geotechnical conditions are the active lateral pressure of the soil 4 backfill and the pressure from the dead weight of the retaining wall 1.

The retaining wall is in stable equilibrium position, and the plane of the supporting prismatic sections 10 are proportional in depth to the ordinates of the lateral pressure and the weight of the retaining wall 1, with the such limiting soil resistance under which it is held in statically balanced position.

Under the conditions of detection the actions of forced horizontal and vertical displacements of the soil, normal loads along the front of movement of the soil are transformed into frontal passive pressure, and in the direction of movement they cause lateral friction, and in all cases they are applied to individual supporting parts 3 at an angle and much larger than the usual active soil pressure.

In this case, an increase in contact pressure at the prismatic support sections 10 above the limiting values in statics is impossible and also leads to the fact that the soils 4, 5 which are imposed on the contact are plastically destroyed and freely move in the cavity 2 until deformation effects are manifested.

The shape of the cavities 2 (truncated pyramid) is most efficient for soil compaction. Soil, falling into the cavity in the region of the lower base of the pyramid and passing the path to its upper base, spontaneously compact. After that, the contact pressure decreases to the initial level, that is, the actual external surface of the retaining wall 1 caused the structure to work with constant resistance to shear forces, impending unstable slopes and slopes, as well as in undermined areas with horizontal and vertical movement of soil.

The current level of computers and software allows for accurate calculations of the stress state of fairly complex systems.

The use of modern programs oriented to the calculation of systems with a large number of unknowns is more efficient than calculations using traditional schemes.

According to some programs, it is possible to solve not only linear but also nonlinear problems.

In this case, consideration is given to the characteristics of the deformation of various materials: reinforced concrete, steel, base, etc.

To account for the work of the soil, several models have been developed that take into account the transition of the soil to a plastic state, viscoelastic deformation, and other models.

For each soil condition and type of stress state, it is advisable to select certain deformation models.

Of particular interest is the PLAXIS program for modelling contact interaction of elements of the "base – engineering structure" system [21, 22].

The PLAXIS program is a finite element package designed specifically for the analysis of the deformation and stability of geotechnical structures.

Modelling using the PLAXIS program made it possible to analyse the stress-strain state of retaining walls interacting with the soil mass in accordance with the design schemes for two options (Fig. 5): option I – corner retaining wall; II option – retaining wall with a structural surface. The simulation results are shown in Fig. 6-8.

As a result, a significantly smaller displacement in the retaining wall with a structural surface $U_{cm}=12.34\times10^{-3}$ m was obtained, but the entire soil mass with a more uniform distribution and reduced values was included on the contact surface σ_{ef} .

In order to determine the forces in the front and foundation plates, two variants of retaining walls were simulated using the "LIRA" program [23]. The design schemes are presented in Fig. 9. The calculations used the Mohr-Coulomb model.

Engineering-geological conditions and design loads are similar to previously performed calculations.

The first stage of the calculation was performed with a surface load q = 49 kN/m². The second stage of the calculation – with an additional vertical component for the front plate from the horizontal movement of the soil q = 21 kN/m².

The obtained values of bending moments in the front and foundation plates for two loads are presented in Fig. 10-12.

A new retaining wall structure with a structural surface was designed to secure the slopes of the Sushkov beam, the area of which is about 2 hectares (Fig. 13, 14).

The beam is located in the central old part of the city of Kryvyi Rih and was formed as a result of iron ore mining.

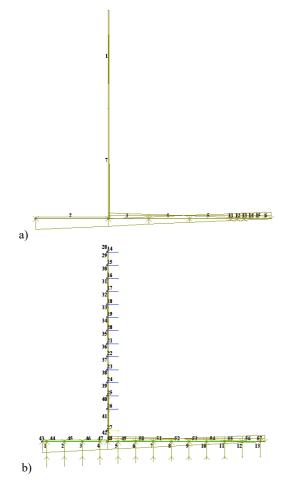


Fig. 9. Settlement schemes: a) I option; b) II option.

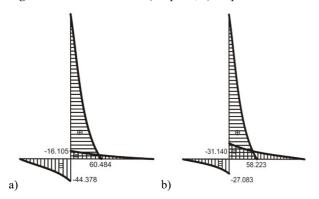


Fig. 10. Diagrams of bending moments: a) I option (first stage); b) II option (first stage).

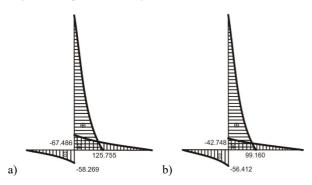


Fig. 11. Diagrams of bending moments: a) I option (second stage); b) II option (second stage).

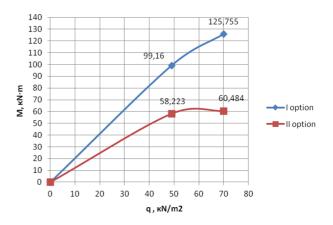


Fig. 12. Dependence of moments on surface load.

Over the past five years, engineering and geological conditions have worsened in the area of the beam: slope shifts in some places up to 12 cm, collapse of some slopes were observed, which in turn affected the normal operation of five-story residential buildings located at a distance of 8 m from the northern border of the beam.

The beam itself has slopes of 5 to 54% and is a dumping site for household waste.

In the geological structure of the site there are loams of the Quaternary sediment, clays, limestones under which the Precambrian clay shales are underlain. Groundwater is at a depth below 21 m.

The rational use of the territory was made possible only through the use of retaining walls with a structural surface.

They were installed around the perimeter of the site and diaphragm along the tiers.

The volumetric work of the retaining walls like the "egg's volume" made it possible to include the entire soil mass around the beam, redistributing the contact forces in the best way, thereby reducing the maximum effective loads from the base.

A compact planning solution allowed in the extreme opposite points of the beam, where the greatest depth is located $-7.5 \text{ m} \div 8 \text{ m}$ (section 2-2, 7-7) place underground parking garages for 100 cars each. At the same time retaining walls simultaneously serve as walls of garages.

Above the underground garage are sports grounds for various sports with a modern bulk coating.

As mentioned above, the slopes of the beam are reinforced with monolithic reinforced concrete retaining walls, the height of which varies from 0.5 m to 3 m (section 1-1, 3-3, 4-4, 5-5, 6-6) depending on the existing depth beams, as well as their tiered device.

The rest of the territory is designed: a quiet recreation area, a children's zone, a decorative zone, a walk-through zone, a public garden area.

5 Conclusions

The validity of the theoretical prediction of the engineering structures' behavior interacting with unevenly deformed foundations cannot be obtained on the basis of the regulatory framework. This gap can be filled in when modeling the "base – engineering structure"

system using modern calculation programs using the finite element method.

The calculation results were confirmed by experimental studies conducted previously. Mathematical modeling made it possible to clearly demonstrate the decrease strain on the contact surface and in the body of the retaining wall with a structural surface with additional forces of soil displacement. The phased filling of voids leads to a uniform distribution of deformations, in the long run increases the life of the structure, thereby ensuring an economic effect. The design feature of the retaining wall with the structural surface during soil interaction with it increases the bearing capacity of the base due to the joint work of the retaining wall structure and the deformable base.

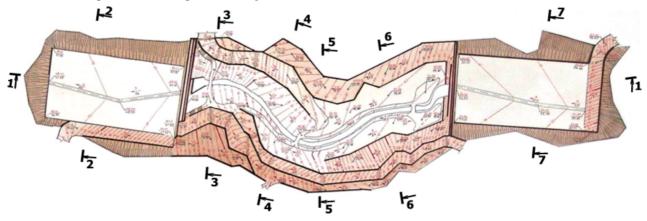


Fig. 13. The plan of the retaining walls with a structural surface in the Sushkov beam.

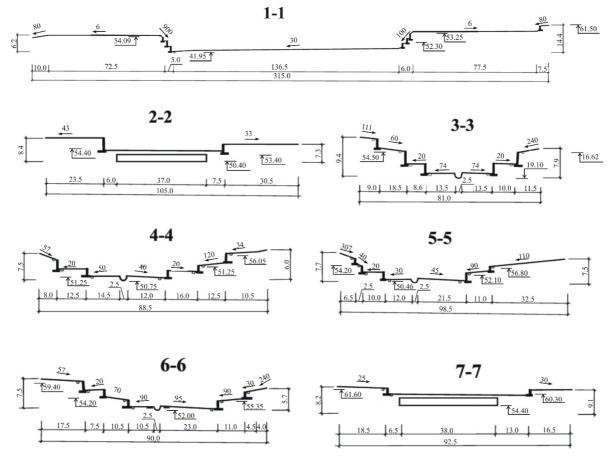


Fig. 14. Sections 1-1, 2-2, 3-3, 4-4, 5-5, 6-6, 7-7 along the Sushkov beam.

The proposed retaining wall can be used to stabilize unstable slopes and inclinations, as well as undermined territories of I, II, III, IV groups and subsidence soils.

The proposed retaining wall is characterized by high reliability of operation in a critical situation of unforeseen

emergency increase in horizontal and vertical power load. This is explained by the work of the structure itself, that is, with increasing load, the supporting areas of the prismatic sections increase all the time, and since the cavity volume sooner or later will be completely filled with soil, then the supporting area of the retaining wall will increase significantly, and the average pressure will decrease when achieved degree of compaction. However, after this, the retaining wall will not be able to work in the mode of redistribution of contact pressures.

With possible vertical displacements of the soil, a constructive solution is also used that allows taking this type of deformation effects into account.

The use of previously unsuitable territory was made possible because of using retaining wall with a structural surface, which made it possible to obtain a significant economic effect due to a new design solution.

The use of this design for securing landslide slopes is advisable when erecting a height of up to 3 m, and the best work is achieved with a perimeter device.

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