Modeling of ultrasonic waves propagation in inhomogeneous medium using fibered spaces method (k-space)

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Abstract

The method for more accurately parameters estimation of the ultrasonic waves propagating in random heterogeneous media, including solid, liquid and gas phases is described

Key words: *phased array, ultrasound, pulp, control*

1. Introduction. For the control of the basic technological parameters and mineral beneficiation process control, an important task is to control the parameters of complex heterogeneous mediums, including solid, liquid and gas phases.

The basic relations describing the ultrasonic oscillations and waves in the medium, follow from the equation of medium state, Newtonian equations of motion and the continuity equation [1-3]. The result are the wave-type equations that can be solved with appropriate initial and boundary conditions.

Let's introduce differential characteristic $I_\lambda(\vec{r},\vec{\Omega})$ to describe the ultrasonic waves radiation field. By $I_\lambda(\vec{r},\vec{\Omega})$ we mean the intensity of the ultrasonic wave (with wavelength λ), which is defined as radiation power per solid angle unit which passing through a unit area perpendicular to the direction $\vec{\Omega}$ of the point $\vec{r}$. Here $\vec{\Omega}$ is the unit vector defining the direction in space, $\vec{r}$ is the radius vector defining the position of a given point in space.

The purpose is to study the volume ultrasonic waves propagation in the gas-containing iron ore slurry, defining of ultrasonic field characteristics in a liquid medium containing solid particles and gas bubbles, influence laws of the suspended in a liquid particles fluctuations on the performance of the ultrasonic field.

The presence of solid particles and gas bubbles introduces some features to the process of ultrasonic wave energy attenuation and scattering [4].

Wave scattering on the solid phase particles becomes significant when the wavelength λ is commensurate with the size of the particles. Consequently, the total intensity of the ultrasonic wave at a given point equal to the sum of the intensities of waves coming from all scattering centers. Scattering cross sections in this case are additive, so the linear absorption and scattering coefficients can be determined by the formulas

$$\Sigma_{-}(\lambda) = n\sigma_C(\lambda),$$
$$\Sigma_S(\lambda) = n\sigma_S(\lambda), \qquad (1)$$

where n - is the particle concentration; $\sigma_{-}(\lambda)$ and $\sigma(\lambda)$ - total cross sections of the acoustic wave absorption and scattering on the particle.

Total cross sections of absorption and scattering depends not only on the wavelength of the ultrasonic vibration, but also on the particle size *r*. The main characteristic of the ultrasonic radiation field $I_\lambda(\vec{r},\vec{\Omega})$ must be determined from the kinematic equation. Before write this equation, we introduce the concept of the differential at the corners scattering coefficient

$$\Sigma_S(\vec{\Omega} \to \vec{\Omega}') = n\sigma_S(\vec{\Omega} \to \vec{\Omega}'), \qquad (2)$$

where $\sigma_S(\vec{\Omega} \to \vec{\Omega}')$ - is differential at the corners energy scattering cross section on the solid phase particle.

The value $\sigma_S(\vec{\Omega} \to \vec{\Omega}')$ represents a part of the energy which scattering by particles in the element of solid angle $d\vec{\Omega}'$. Obviously, the total scattering cross section σ_s associated with differential scattering cross section ratio

$$\sigma_S = \int_{4\pi} \sigma_S(\vec{\Omega} \to \vec{\Omega}')d\vec{\Omega}', \qquad (3)$$

The kinetic equation which is solved by function $I_\lambda(\vec{r},\vec{\Omega})$ can be obtained by considering the energy balance in a volume element of the phase space

$$\vec{\Omega}\nabla I_\lambda(\vec{r},\vec{\Omega}) = -\Sigma(\lambda) I_\lambda(\vec{r},\vec{\Omega}) + \\ + \int d\Omega' \Sigma_S(\vec{\Omega}' \to \vec{\Omega}) I_\lambda(\vec{r},\vec{\Omega}') + S_\lambda(\vec{r},\vec{\Omega}), \qquad (4)$$

where $\Sigma(\lambda) = \Sigma_{\_}(\lambda) + \Sigma_S(\lambda)$. $S_\lambda(\vec{r},\vec{\Omega})$ - is the ultrasound source radiation density function, which determines the average amount of energy emitted per unit time single phase volume.

Under the phase coordinates means the totality of variables r and Ω, while elementary phase volume is determined by the product $d\vec{r} \cdot d\vec{\Omega}$.

Equation (4) can be reduced to an integral equation of the form (5)

$$I_\lambda(\vec{r},\vec{\Omega}) + \int d\vec{r}' \int d\vec{\Omega} \sum\nolimits_s (\vec{\Omega}' - \vec{\Omega}) \frac{e^{-\tau(\vec{r}',\vec{r},\lambda)}}{|\vec{r}-\vec{r}'|} \times$$
$$\times\, \delta\left[\vec{\Omega} - \frac{(\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|}\right] I_\lambda(\vec{r}',\vec{\Omega}') + I_\lambda^0(\vec{r},\vec{\Omega}) \quad (5)$$

where $\tau(\vec{r}',\vec{r},\lambda) = \Sigma(\lambda)|\vec{r}-\vec{r}'|$, $\delta(\cdot)$ is the Dirac delta function; $I_\lambda^\circ(\vec{r},\vec{\Omega}) = \int_0^\infty S_\lambda(\vec{r}-\xi\vec{\Omega},\vec{\Omega}) e^{-\tau(\xi,\lambda)} d\xi$ - is the free term of the integral equation (5), which determines the intensity of the unscattered ultrasonic wave; $\xi = |\vec{r}-\vec{r}'|$.

The solution of equation (5) can be written in the form of a Neumann series [5], which is the expansion of the ultrasonic waves scattering multiplicity solution.

However, to obtain an expression analytically even for the singly scattered radiation is impossible. Therefore it is necessary to apply numerical methods for solving integral equations of the form (5).

Let's consider the method of fiber spaces (k-space) for modeling of ultrasonic wave propagation in inhomogeneous medium using coarse grids, with maintaining the required accuracy [6 -8].

We describe the ultrasonic waves propagation depending on the mass conservation equations, momentum conservation law and the equation of state using the first order dual equations, which can be summarized as follows [8, 9]

$$\frac{\partial p(\bar{x},t)}{\partial t} + \rho(\bar{x})c^2(\bar{x})\nabla v(\bar{x},t) = -\alpha(\bar{x})\,p(\bar{x},t), \quad (6)$$

$$\rho(\bar{x})\frac{\partial v(\bar{x},t)}{\partial t} + \nabla p(\bar{x},t) = 0, \quad (7)$$

where $p(\bar{x},t)$- the time and space dependent ultrasound pressure perturbations (x - 3D Cartesian axis (x, y, z)); $\rho(\bar{x})$- is the spatially dependent density; $c(\bar{x})$- is the spatial dependent sound speed; $v(\bar{x},t)$- is the velocity of the particle and $\alpha(\bar{x})$- is the absorption coefficient which equivalent to the inverse of the relaxation time.

Let's represent the all absorption effects with one relaxation time. From (7), the simplified equation can be written as follows

$$\frac{\partial v(\bar{x},t)}{\partial t} = \frac{-\nabla p(\bar{x},t)}{\rho(\bar{x})}$$

We differentiate (6) with respect to time and variations in (7), and the final equation can be represented as follows

$$\frac{\partial^2 p(\bar{x},t)}{\partial t^2} + \rho(\bar{x})c^2(\bar{x})\frac{\partial}{\partial t}\nabla\partial v(\bar{x},t) = -\alpha(\bar{x})\frac{\partial p(\bar{x},t)}{\partial t}, \quad (6')$$

$$p(\bar{x},t)\frac{\partial}{\partial t}\nabla v(\bar{x},t) + \frac{\partial v(\bar{x},t)}{\partial t}\nabla\rho(\bar{x}) + \nabla^2 p(\bar{x},t) = 0, \quad (7')$$

Taking into account the permutations (6')

$$\frac{\partial}{\partial t}\nabla v(\bar{x},t) = -\left(\frac{\alpha(\bar{x})}{\rho(\bar{x})c^2(\bar{x})}\frac{\partial p(\bar{x},t)}{\partial t} + \frac{1}{\rho(\bar{x})c^2(\bar{x})} + \frac{\partial^2 p(\bar{x},t)}{\partial t^2}\right)$$

By substituting this equation in (7'), we obtain

$$\frac{-\alpha(\bar{x})}{c^2(\bar{x})}\frac{\partial p(\bar{x},t)}{\partial t} - \frac{\partial^2 p(\bar{x},t)}{c^2(\bar{x})\partial t^2} - \frac{1}{\rho(\bar{x})}\nabla p(\bar{x},t)\nabla p(\bar{x}) + \nabla^2 p(\bar{x},t) = 0, \quad (8)$$

The simplification of the pressure deviation to the density gradient can be represented as follows

$$\nabla\left(\frac{\nabla p(\bar{x},t)}{\rho(\bar{x})}\right)=\frac{\nabla^2 p(\bar{x},t)}{\rho(\bar{x})}-\frac{\nabla p(\bar{x},t)\nabla p(\bar{x})}{\rho(\bar{x})^2}, \qquad (9)$$

Taking into account (8), eq. (9) can be represented as follows

$$\nabla\left(\frac{1}{\rho(\bar{x})}\nabla p(\bar{x},t)\right)-\frac{1}{\rho(\bar{x})c^2(\bar{x})}\frac{\partial^2 p(\bar{x},t)}{\partial t^2}=\frac{\alpha(\bar{x})}{\rho(\bar{x})c^2(\bar{x})}\frac{\partial p(\bar{x},t)}{\partial t}, \qquad (10)$$

This is a linear wave equation of ultrasonic wave propagation in the heterogeneous medium with the absorption parameters.

Let's simplify (10) by separating the parameters of the sound velocity $c(\bar{x})$ and density $\rho(\bar{x})$ from the second derivatives of pressure taking into account the spatial and temporal variables to solve the problem of ultrasoundpropagation using the fiber space method.

The original equation can be written in the form

$$\nabla\left(\frac{1}{\rho(\bar{x})}\nabla p(\bar{x},t)\right)-\frac{1}{\rho(\bar{x})c^2(\bar{x})}\frac{\partial^2 p(\bar{x},t)}{\partial t^2}=0, \qquad (11)$$

The normalized pressure can be represented as follows

$$\psi(\bar{x},t)=\frac{p(\bar{x},t)}{\sqrt{p(\bar{x})}}$$

By substituting this equation in (11) we obtain

$$\nabla\left(\frac{1}{\rho(\bar{x})}\nabla p^{1/2}(\bar{x},t)\psi(\bar{x},t)\right)=\frac{\rho^{1/2}(\bar{x})}{\rho(\bar{x})c^2(\bar{x})}\frac{\partial^2\psi(\bar{x},t)}{\partial t^2}$$

After simplifying

$$\nabla^2\psi(\bar{x},t)-\rho^{1/2}(\bar{x})\psi(\bar{x},t)\nabla^2\rho^{1/2}(\bar{x})=\frac{1}{c^2(\bar{x})}\frac{\partial^2\psi(\bar{x},t)}{\partial t^2}$$

Taking into account further simplifications the equation takes the form

$$\nabla^2\psi(\bar{x},t)-\frac{1}{c_0^2}\frac{\partial^2\psi(\bar{x},t)}{\partial t^2}=\frac{1}{c_0^2}\left[c_0^2\rho^{1/2}(\bar{x})\left(\nabla^2\rho^{1/2}(\bar{x})\right)\psi(\bar{x},t)+\left(\frac{c_0^2}{c^2(\bar{x})}-1\right)\frac{\partial^2\psi(\bar{x},t)}{\partial t^2}\right]$$

Even more simplification can be obtained by determining the functions $q(r, t)$ and $v(r, t)$efficient sources, which can be summarized as follows

$$q(\bar{x},t)=c_0^2\rho^{1/2}(\bar{x})\psi(\bar{x},t)\nabla^2\rho^{-1/2}(\bar{x})$$

$$v(\bar{x},t)=\left(\frac{c_0^2}{c^2(\bar{x},t)}-1\right)\psi(\bar{x},t)$$

By simplifying (11) we obtain

$$\nabla^2\psi(\bar{x},t)-\frac{1}{c_0^2}\frac{\partial^2\psi(\bar{x},t)}{\partial t^2}=\frac{1}{c_0^2}\left(q(\bar{x},t)+\frac{\partial^2 v(\bar{x},t)}{\partial t^2}\right), \qquad (12)$$

This equation can be easily transformed into the frequency domain by using the three-dimensional spatial Fourier transform as follows

$$k^2F(\mathrm{k,t})-\frac{1}{c_0^2}\frac{\partial^2F(\mathrm{k,t})}{\partial \mathrm{t}^2}=\frac{1}{c_0^2}\left(Q(\mathrm{k,t})+\frac{\partial^2V(\mathrm{k,t})}{\partial \mathrm{t}^2}\right), \qquad (13)$$

where *F(k,t), Q(k,t)* и *V(k,t)* – three-dimensional spatial Fourier transformation of values $\psi(\overline{x},t)$, $q(\overline{x},t)$ and $v(\overline{x},t)$ respectively. Equation (13) satisfies the total wavefield, and is defined as the sum of the incident and scattered field $\psi(\overline{x},t)=\psi_i(\overline{x},t)+\psi_s(\overline{x},t)$, and the scattered wave field.

$$\nabla^2\psi(\overline{x},t)-\frac{1}{c_0^2}\frac{\partial^2\psi(\overline{x},t)}{\partial t^2}=0$$

For the case of an inhomogeneous medium, we introduce an additional source $w(\overline{x},t)=\psi_s(\overline{x},t)+v(\overline{x},t)$ and by substituting it into (13) we obtain the following expression

$$\frac{\partial^2W(k,t)}{\partial t^2}=k^2c_0^2\left[W(k,t)-V(k,t)\right]-Q(k,t), \qquad (14)$$

where $V(k,t)=\mathbf{F}\left[\left(1-\frac{c^2(\overline{x})}{c_0^2}\right)\left(\psi_i(\overline{x},t)+w(\overline{x},t)\right)\right]$

$$Q(k,t)=c_0^2\mathbf{F}\left[\sqrt{\rho(\overline{x})}\nabla^2\rho^{1/2}(\overline{x})=\left[\psi_i(\overline{x},t)+w(\overline{x},t)-v(\overline{x},t)\right]\right]$$

where F - is a spatial Fourier transform.

Let's use the substandard finite difference approach to solve this equation [7]. Discretization of the time derivative gives

$$W(k,t+\Delta t)-2W(k,t)+W(k,t-\Delta t)=4\sin^2\left(\frac{c_0k\Delta t}{2}\right)\times$$
$$\times\left[V(k,t)-W(k,t)-\frac{Q(k,t)}{c_0^2k^2}\right], \qquad (15)$$

Consider the wave equation on the gray scale for the fiber space method (*k*-space), which includes the non-linear characteristic of ultrasound, which can be represented as follows [10]:

$$\nabla^2\psi(\overline{x},t)-\sqrt{\rho(\overline{x})}\psi(\overline{x},t)\nabla^2\frac{1}{\sqrt{\rho(\overline{x})}}-\frac{1}{c^2(\overline{x})}\frac{\partial^2\psi(\overline{x},t)}{\partial t^2}=-\frac{\beta(\overline{x})}{\sqrt{\rho_0}c_0^4}\frac{\partial^2\psi^2(\overline{x},t)}{\partial t^2}$$

where $\psi^2(\overline{x},t)$ - is the nonlinearity source, $\beta(\overline{x})$ - is the nonlinearity coefficient. The harmonic oscillations equation can be represented as follows

$$\frac{\partial^2W2(\overline{k},t)}{\partial t^2}=\left(c_0^2k^2\right)\left(VNL2(\overline{k},t)-W2(\overline{k},t)\right)-Q(\overline{k},t), \qquad (16)$$

where $w2(\overline{x},t)=\psi_s(\overline{x},t)+v_{NL2}(\overline{x},t)$ – additional source; $W2(\overline{k},t)$ - is a spatial Fourier transform.

$$v_{NL2}\left(\overline{x},t\right)=\left(\frac{c_0^2}{c^2\left(\overline{x}\right)}-1\right)\psi\left(\overline{x},t\right)-\frac{\beta\left(\overline{x}\right)}{\sqrt{\rho_0}c_0^2}\left(\psi_s^2\left(\overline{x},t\right)+2\psi_s\left(\overline{x},t\right)\psi_i\left(\overline{x},t\right)\right)$$

After the spatial Fourier transformation the equation can be expressed as follows

$$VNL2\left(\overline{k},t\right)=$$

$$=\mathbf{F}\left\{\begin{array}{l}\left(\dfrac{c_0^2}{c^2\left(\overline{x}\right)}-1\right)\left[\psi_i\left(\overline{x},t\right)+w2\left(\overline{x},t\right)-\dfrac{\beta\left(\overline{x}\right)}{\sqrt{\rho_0}c_0^2}\left(\psi_s^2\left(\overline{x},t\right)-2\psi_s\left(\overline{x},t\right)\psi_i\left(\overline{x},t\right)\right)\right.\\ -\dfrac{\beta\left(\overline{x}\right)}{\sqrt{\rho_0}c_0^2}\left(\psi_s^2\left(\overline{x},t\right)-2\psi_s\left(\overline{x},t\right)\psi_i\left(\overline{x},t\right)\right)\end{array}\right\}$$

$$Q\left(k,t\right)=\mathbf{F}\left[c_0^2\sqrt{\rho\left(\overline{x}\right)}\nabla^2\left(\frac{1}{\sqrt{\rho\left(\overline{x}\right)}}\right)\left[\psi_i\left(\overline{x},t\right)-w2\left(\overline{x},t\right)-v_{NL2}\left(\overline{x},t\right)\right]\right]$$

The introduction of the nonlinearity term in fiber space method makes it easier to calculate the actual relief temperature in heterogeneous large scale models.

Conclusions. To build a model of the ultrasonic field in a randomly inhomogeneous medium, the fiber spaces method (*k*-space), which increased the accuracy of parameter estimation field is used.

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