Simulation of high-energy ultrasound propagation in heterogeneous medium using k-space method

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Abstract
The method and software based on this method for parameters estimation of the ultrasonic waves propagating in random heterogeneous medium are described
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Control of mineral beneficiation process requires controlling the parameters of complex heterogeneous mediums, including solid, liquid and gas phases.

Ultrasonic oscillations are periodic disturbances of the elastic medium state, characterized by a change in its physical properties, which occur synchronously with perturbation. During ultrasonic propagation the local medium volume oscillations are transmitted to adjacent areas by means of elastic waves, characterized by a change in medium density in space and which transfer the fluctuations energy [1-3].

$$\rho_0 \frac{\partial u}{\partial t} + \nabla p = -\rho \frac{\partial u}{\partial t} - \frac{1}{2}\rho_0 \nabla u^2, \quad \frac{\partial \rho}{\partial t} + \nabla \cdot (\rho_0 u) = -\nabla \cdot (\rho u) \qquad (1)$$

where p is the acoustic pressure, ρ is the density, u is the particle velocity, ρ_0 is the ambient density. The second order terms in (1) can be re-written in terms of the Lagrangian density by the repeated

$$\frac{\partial u}{\partial t} = -\frac{1}{\rho}\nabla p, \quad \frac{\partial \rho}{\partial t} = -\rho_0 \nabla \cdot u - u \cdot \nabla \rho_0 + \frac{1}{\rho_0 c_0^4}\frac{\partial p^2}{\partial t} \qquad (2)$$

Let's neglect thermoviscous losses and include a phenomenological loss operator to account the arbitrary power law absorption [8],

$$p = c_0^2 \left(\rho + d \cdot \nabla \rho_0 + \frac{B}{2A\rho_0}\frac{\rho^2}{\rho_0} - \tau \frac{\partial}{\partial t}\left(-\nabla^2\right)^{\frac{y}{2}-1}\rho - \eta \left(-\nabla^2\right)^{\frac{y+1}{2}-1}\rho \right) \qquad (3)$$

where c_0 is the isentropic sound speed, d is the particle displacement vector, and τ and η are the absorption and dispersion proportionality coefficients where $\tau = -2\alpha_0 c_0 y - 1$ and $\eta = 2\alpha_0 c_0 y \cdot \tan(\pi y/2)$. α is the acoustic absorption where $\alpha = \alpha_0 \omega y$, α_0 is the absorption coefficient prefactor and y is the power law exponent [9].

The expressions given in (2) and (3) are the acoustic particle velocity, density, and pressure coupled set of equations.

Let's combine these expressions into a single second order wave equation for the acoustic pressure. The modified Westervelt equation by neglecting higher order absorption, nonlinearity, and heterogeneity terms, can be written in the following form:

The basic relations describing ultrasonic oscillations and waves in the medium, follow from the equation of medium state, Newtonian equations of motion and the continuity equation [4,5]. The results are the wave-type equations that can be solved with appropriate initial and boundary conditions.

The equations of ultrasonic waves nonlinear propagation in heterogeneous medium can be derived from the mass, momentum, and energy conservation laws under the assumption of a quiescent, isotropic, and inviscid medium as follows[6,7].

substitution of the acoustic equations in linearized form [8]. The Lagrangian density terms can be neglected as follows:

then the equation of state from the total pressure expansion using a Taylor series can be written as follows:

$$\nabla^2 p - \frac{1}{c_0^2}\frac{\partial^2 p}{\partial t^2} - \frac{1}{\rho_0}\nabla \rho_0 \cdot \nabla p + \frac{\beta}{\rho_0 c_0^4 \partial t^2} +$$
$$+ \left(\tau \frac{\partial}{\partial t}\left(-\nabla^2\right)^{\frac{y}{2}} + \eta \left(-\nabla^2\right)^{\frac{y+1}{2}} \right) p = 0$$
$$(4)$$

where $\beta = 1 + B/2A$ is the nonlinearity coefficient.

By neglecting higher order absorption and nonlinearity effects, the conservation equations in (2) using a k-space method [10] can be written as follows

$$\frac{\partial}{\partial \xi}p^n = \mathbb{F}^{-1}\left\{ik_\xi \kappa \mathbb{F}\{p^n\}\right\}$$

$$u_\xi^{n+1} = u_\xi^n - \frac{\Delta t}{\rho_0}\frac{\partial}{\partial \xi}p^n \qquad (5)$$

$$\frac{\partial}{\partial \xi}u_\xi^{n+1} = \mathbb{F}^{-1}\left\{ik_\xi \kappa \mathbb{F}\{u_\xi^{n+1}\}\right\}$$

$$\rho_\xi^{n+1} = \rho_\xi^n - \Delta t \rho_0 \frac{\partial}{\partial \xi} u_\xi^{n+1} + \frac{(p^n)^2 - (p^{n-1})^2}{N\rho_0 c_0^4}$$

where i is the imaginary unit, k_ξ is the wavenumber in the ξ direction, κ is the k-space adjustment where $\kappa = sinc(c_0 k \Delta t/2)$, F and F^{-1} denote the forward and inverse Fourier transform, Δt is the time step.

$$L = -\tau F^{-1}\left\{(k)^{y-2} \mathbb{F}\left\{\rho_0 \sum_\xi \frac{\partial}{\partial \xi} u_\xi^{n+1}\right\}\right\} + \eta F^{-1}\left\{(k)^{y-1} \mathbb{F}\{\rho^{n+1}\}\right\} \quad (7)$$

Let's simulate the ultrasonic pressure field propagation in a heterogeneous medium using k-Wave toolbox (Matlab) which is designed for time domain ultrasound simulations in complex media like heterogeneous pulp. The simulation functions of this software are based on the k-space method and are both fast and easy to use [10].

The net pressure of all piezoelectric elements can be obtained by adding the effects of each source and written in the form

$$P_{net}(x, y, z) = \sum_{i=1}^n p_i(x, y, z) \quad (8)$$

Due to attenuation, the useful power at the point (x, y, z) is given by [11]

$$q(x, y, z) = \frac{\alpha P_{net}^2(x, y, z)}{\rho c} \quad (9)$$

The total energy at a point (x, y, z) is given by

$$I(x, y, z) = \frac{p^2(x, y, z)}{2\rho c} \quad (10)$$

where $I(x, y, z)$ - intensity at the point (x, y, z), W/m^2.

The results of the ultrasonic wave propagation through a heterogeneous medium with density $\rho = 1500\,\text{kg/m}^3$, for source strength of 1MPa and tone burst frequency of 1MHz for 16-
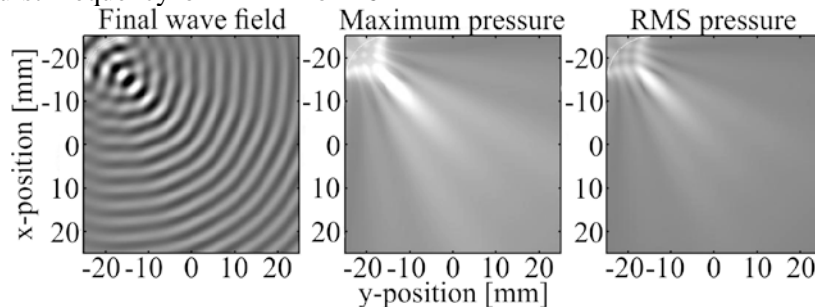
The corresponding equation of state in discrete form is

$$p^{n+1} = c_0^2\left(\rho^{n+1} + \frac{B}{2A}\frac{1}{\rho_0}(\rho^{n+1})^2 - 1\right) \quad (6)$$

where the total density is given by $\rho = \sum_\xi \rho \xi$ and the discrete loss term is

element phased array with focus distance of 20mm are shown in Fig. 1. The central slice absorption distribution in grayscale as a background and the square of the pressure distribution on the surface of this background are shown.
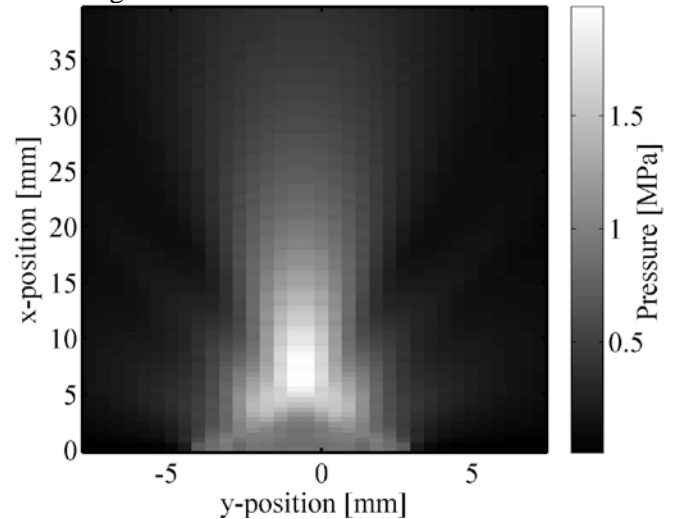

Figure 1 Total beam pattern using maximum of recorded pressure.

The final pressure field (a), the maximum pressure (b) and standard pressure (c) of the beam are shown on Fig. 2. The transducer focus and sidelobes are visible.
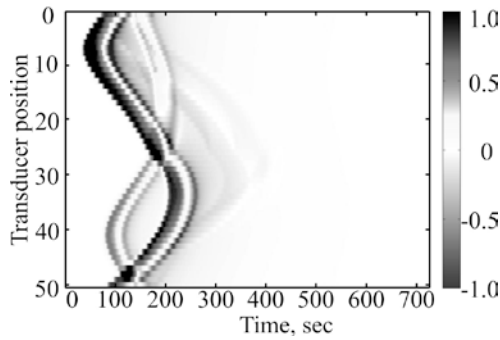

Figure 2 Ultrasonic wave propagation in heterogeneous medium: a) the final pressure field, b) the maximum pressure c) the rms pressure

Figure 3 The shape of the main wavefront.

The linear cross-section of the focus in *x* direction is shown on Fig. 4: 1) for the single source; 2) simulation by *k*-space method in the water; 3) in a heterogeneous medium.
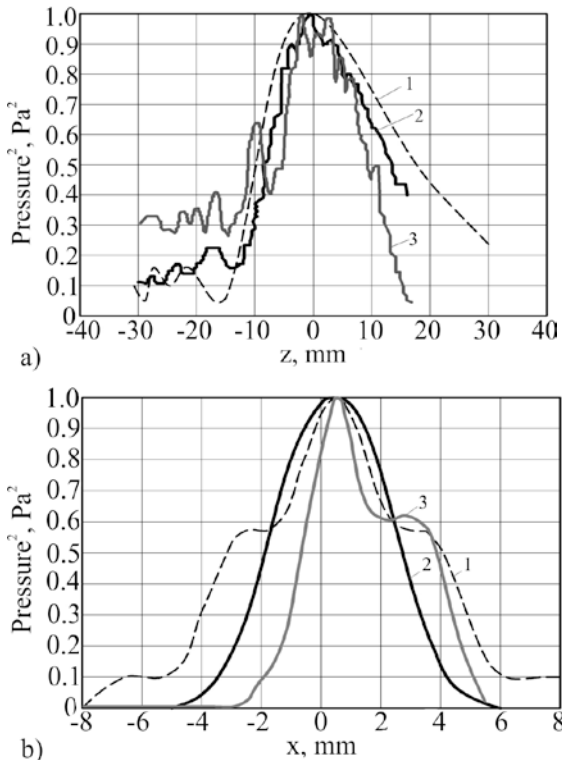

Figure 4 The simulation results comparison of the normalized square of pressure for: 1) a simple screened source, 2) modeling by *k*-space method in a homogeneous medium (water) and 3) in the inhomogeneous medium (pulp) along the axis: a) - *z* and b) - *x*.

For the process of energy accumulation and transfer in a certain point of space can be given a numerical estimate by measuring the temperature. The simulation of temperatures was performed using the heat transfer equation [12]. The accumulated power was extracted from the three perpendicular lines that crossed the values of the simulated and measured ultrasound pressure focal area change (Fig. 5).
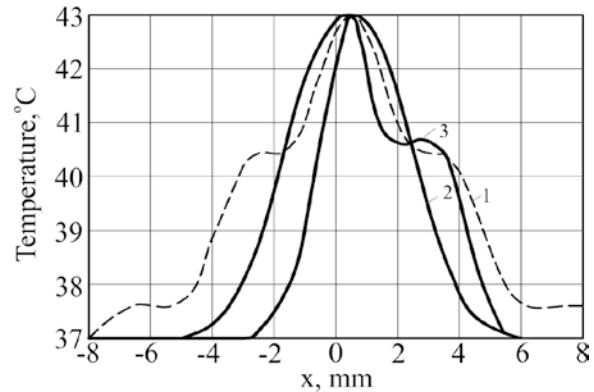

Figure 5 The simulation results comparison of the temperature distribution for: 1) a simple screened source, 2) modeling by *k*-space method in a homogeneous medium (water) and 3) in the inhomogeneous medium (pulp) along the *x* axis.

Conclusions

To build a model of the ultrasonic field in a randomly inhomogeneous medium, the fiber spaces method (*k*-space), which increased the accuracy of parameter estimation field is used.

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