MATHEMATICAL MODELING OF A ROTARY DRILLING RIG DRILLING ROD TRANSVERSE OSCILLATIONS

Anatolii.S. Gromadskyi,
Kryvyi Rih National University, Doctor of Sciences (Engineering), Professor, Ukraine

Vladyslav.A. Gromadskyi,
Kryvyi Rih National University, Candidate of Sciences (Engineering), associate professor, Ukraine

Viktor.A. Gromadskyi,
Kryvyi Rih National University, Candidate of Sciences (Engineering), lecturer, Ukraine

Oleksii.Yu. Krivenko,
Kryvyi Rih National University, Candidate of Sciences (Engineering), associate professor, Ukraine

The operation of drilling rigs in the modes of increased vibration of drill rods increases the number of main unit breakdowns, increases the wear of the rods, reduces the technical and economic parameters of drilling, worsens the working conditions of operating personnel. Therefore, the choice of rational modes of rotary drilling rigs operation, which allow reducing oscillations of drill rods, is an actual problem that meets the requirements of their operation. Many reasons affecting the vibration resistance of the operation of rotary drilling rigs, indicate the expediency in the study to apply modern methods, which include the method of mathematical modeling. The paper considers the developing of a mathematical model of the transverse oscillations of the drill rod, taking into account the principal of physics. The development of such a model made it possible not only to distinguish the natural frequencies of the transverse oscillations of the drill rod, but also to obtain an analytical formula that relates the values of these frequencies to the parameters characterizing the operation of the drill rod. It is important that this formula managed to be presented in a dimensionless form, combining the parameters into complexes, which reduced the number of variables in the formula from five to two, thereby facilitating the study.

The problem and its connection with practical tasks.

Rotary drilling rigs are widely used when minerals are open-mined. The functioning of rotary drilling rigs in modern conditions is associated with the implementation of forced modes, which leads to increased vibration of the drill rods, related, in particular, to their transverse oscillations. Forcing the drilling mode, associated with the need to increase the productivity of drilling machines, in turn, leads not only to an increase in the oscillation of the drill rods, but also to the expansion of its spectral composition. The operation of machines in the modes of increased vibration of the drill rods leads to an increase in breakdowns of the main components, increased wear of the rods, a decrease in technical and economic indicators of drilling, deterioration of the working conditions of the staff. The choice of rational modes of rotary drilling rigs operation, which will reduce the oscillations of the drill rods, is an actual problem related to their operating conditions.

Therefore, by mathematical modeling of nonlinear dimensionless dependencies of frequencies and amplitudes of transverse natural oscillations of a drill rod, taking into account the physical laws associated with this process, it will further allow to study the effect of parameters not separately from the amplitudes of forced transverse oscillations, but in combination and reduce adverse conditions of the machine as a whole, is an urgent task and meets the requirements of their operation.

The simulation of oscillations of a drilling rod during the operation of a rotary drilling rig involves two stages. At the first stage, mathematical modelling of drilling rig oscillations is carried out, based on the corresponding equations of mathematical physics, the basis of which are the conservation laws. Naturally, mathematical modelling is an idealization of the studied processes.

Therefore, to establish the adequacy of solutions obtained by mathematical modeling, it is necessary to introduce the second stage, namely, testing on real objects. At the same time, such tests on a drilling rod under real conditions are impossible, since they lead to violations of the technological regimes of drilling and, as a result, to significant economic costs. As one of the possible ways to overcome this problem is a computer modeling of a drilling rod with the help of which the necessary experiments can be carried out.

Analysis of research and publications.

According to the author of the paper opinion [1], the main cause of vibration of rotary drilling rigs when blastholes drilling in open mining is the elastic oscillations of the drilling rod. Tensometric studies of stresses in the drilling rod have shown that the axial force at the bottom of the borehole is not constant; it changes periodically, with a constant component and a variable, the amplitude of which is approximately 40% of the constant component. Similar oscillatory phenomena are inherent in the torque on the drilling rod, where the variable component reaches 70-75% of the constant. The nature of the stress variation in the bar of the drilling rod is given in paper [1], which shows the oscillogram of mechanical stresses in the drilling rod bar at the speed n=50 min^{-1} and the axial force P=225 kN. In this case, the resulting stresses in the bar have the form: $\sigma=\sigma_{cm}+\Delta\sigma\cdot\sin\omega_1\cdot t$ - compression stress, $\tau=\tau_{cm}+\Delta\tau\cdot\sin\omega_2\cdot t$ - torsional stresses (tangential). Experimental studies conducted at the Central mining combine «Uralasbest» (inclined drilling machine 2RDR-200N) showed that the frequency of drilling increases with increasing speed of rotation of the drilling tool. When changing the rotational speed from 50 to 150 min^{-1}, the oscillation frequency of the variable component of the axial force is in the range of 0,8-1,2 Hz, the frequency of the variable component of the torque varies from 1,2 to 2 Hz. The nature of drilling rod fluctuations is even more complex. The use of natural vibrations of rod in rotary drilling rigs in open mining in order to increase their productivity requires the development of the issue of their vibration isolation. The authors declare that one of the ways to solve this issue is to replace the rigid connection of the drilling rod with the rotary drilling rig by an elastic connection with a relatively soft characteristic. However, the authors [1] do not provide any calculations or recommendations regarding the characteristics of the elastic connections of the drilling rod with the rotary drilling rig. While we [2], on the basis of calculations, have found that the cause of intense longitudinal oscillations of the rotator and drilling rod of rotary drilling rig is the resonant vibration of the rotator suspension on spring-damped cable pull rods of polyspast pulley block feedings to the bottomhole and the polyspast pulley blocks of the drilling rod removal from the drilled borehole. Since the polyspast pulley blocks of feed and removal of bar are pretensioned, the rotator suspension from the drilling rod is constantly spring-loaded from both sides by the ropes of the polyspast pulley blocks and has different natural frequencies of oscillations depending on the number of bars in the drilling rods. So, for example, the suspension of the rotator of the USBSH-250A machine with one weighed bar with a diameter of 219 mm and a length of 8000 mm has its own oscillation frequency f=8.8 Hz, with two screwed-on rods f=7.7 Hz, with three screwed rods -f=6.9 Hz.

Saroyan A.E. carried out research at the drilling of deep boreholes 2200-2500 m [3]. He notes that with the continuous contact of the drilling bit teeth with the borehole bottom in the string, elastic waves arise associated with both rolling of the drill bit roller cutter from a tooth onto a tooth, and with the rolling of the drill bit roller cutters like cones along a wavy of the face. However, how this waviness and the type of this waviness are recorded is not shown by the author.

In [3], it was also shown that another source of longitudinal vibrations is the rotation of the drill string. Heterogeneity of drilled rocks, changes in friction forces along the boreholes and other causes lead to uneven rotation of the drill string.

In [4], for the first time for rotary drilling rigs, the authors give a mathematical model of the longitudinal and torsional vibrations of a drill rod, as well as the dependence of the low frequencies of their own longitudinal and torsional vibrations on the length of the drill rods for the SBSH-250N and SBSH-250 MNA drilling rigs. It is shown that with increasing of borehole depth by increasing the drilling rod, natural frequencies of, both longitudinal, transverse and torsional vibrations of the rods decrease according to a non-linear law, while approaching each other in magnitude.

Simonov V.V., Yunin Ye.K. [5] and Saroyan A.Ye. [3] agree, on the basis of the tests carried out, the falling torque characteristic of the drill bit with increasing angular velocity of its rotation causes torsional and longitudinal vibrations during the drilling of deep (up to 3000 m) boreholes.

Sukhanov A.F., Kutuzov B.N., Schmidt R.G. [7] note that with an increase in the bit rotation speed, the vibration parameters also grow, and when drilling fragile monolithic rocks less intensively than when drilling strong, fractured rocks. With an increase in the speed of rotation of the drill rod over 120-150 min^{-1}, in drill rods with the cable-polyspast supply system, resonant phenomena often occur, precluding the further possibility of operating without changing the operating parameters. However, in the paper there is no data on the value of the frequencies and amplitudes of these resonant oscillations. In the paper [7] it is also indicated that when the rotary drilling rigs operate, the axial feed forces and the number of rotations of the rotator can reach such values at which the drilling rod loses stability. The authors have performed a theoretical definition of the vibration resistance of drill rods, as a result of which it is shown (Fig. 17, p. 48 [7]) that for rods of 10 m long with an outer diameter of

Ø 152 mm and a wall thickness h = 12 mm with an axial force P = 200 kN stability is lost when the number of rotations n=275 min^{-1}, and for P=500 kN - when n=80 min^{-1}. Experimental data (Fig. 20, p. 53 [7]), given by the authors on the bar Ø 152 mm, length 6 m (that is, more rigid than 10 m), obtained on the SBSH-200 drilling rig with a rigid cartridge feed scheme show that stability is lost when P=180-190 kN and the number of rotations n=150 min^{-1}, does not confirm the theory set forth in the same source, since the rigid bar loses stability with less by 5-10% efforts and 54.5% less rotations than according to theoretical calculations. In addition, it should be noted that currently the most common are the more powerful SBSH-250 MNA-32 drilling rigs, which use 8-meter thick drill rods ø215×51.5 and ø203×50. At the same time, in iron-ore open pits, for example, in the Kryvyi Rih iron-ore basin, they drill to a depth of up to 24 m, that is, with no more than 3 rods at a speed of n≤110 min^{-1} and supply efforts of P≤220 kN. In the well-known publications there are no data on the stability of such drill rods under these drilling conditions. In a later paper [17] Kutuzov B.N. with co-authors gave data on the loss of stability of the drill rod of the SBSH-250MN drilling rig P≥250 kN and the number of rotation n≥120 min^{-1}, taking into account the centrifugal force arising from the rotation of the axially bent drilling rod. Will a loss of stability of the drill rod occur, taking into account the centrifugal force arising from the rotation of the axially curved drill rod up to 24 m long under the conditions of the Kryvyi Rih iron ore basin, is unknown. To clarify these contradictions, it is necessary to carry out special studies on the stability of the drill rod in the range of real axial feed forces during drilling with rotary drilling rigs of the SBSH-250 type.

In paper [8], the design model is presented and the natural frequencies of the transverse oscillations of the drill rod are defined below -ω=61-59 1/s for various axial feed forces R in the range of 50-350 kN for the SBSH-250N machine of the Novokramatorsk plant without specifying the rod parameters. This paper presents recommendations for rotational frequencies of the drill rod, taking into account the forced resonant oscillations for the SBSH-250N drilling rigs with one rod equipped with a roller cutter with a number of teeth z=3.

However, the SBSH-250N drilling rig was a prototype that was not put into production. Therefore, it is not possible to verify the recommendations of the authors.

When describing the operating conditions of rotary drilling rigs, most of the authors, as the main source of vibration, define the frequency of rotation of the drill rod, longitudinal, torsional and transverse oscillations of the drilling rod pipe strings. However, there are no data on the amplitudes of these oscillations and the conditions for their transmission to the drilling rig, which makes it impossible to predict in advance the vibration of the bar and the whole rig and consciously control the modes of its operation, reduce or completely eliminate the extreme oscillations, which, as mentioned above, lead to the destruction of the metal structure masts and significant vibration in the workplace of the operator of the drilling rig.

Analysis of the processes that determine the transverse oscillations of the drill rods indicates both the number of them and the impossibility of taking them all into account. Therefore, we consider it expedient to carry out mathematical modeling of the transverse oscillations of the drill rod, taking into account the physical laws associated with these processes

Material presentation and findings.

In modeling, the drill rod is considered as a hollow rod of circular cross section, both ends of which are floated [9]. In this case, the bar takes compression from the feed force.

According to the scheme of the drill rod, shown in Figure 1, a mathematical model describing its transverse oscillations can be represented as a homogeneous partial differential equation [9]

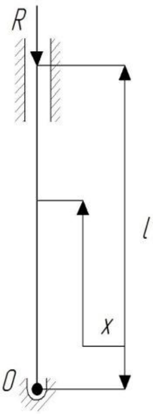

Fig.1. Drill rod scheme

$$EJ\frac{\partial^4 y}{\partial x^4} + R\frac{\partial^4 y}{\partial x^4} + m\frac{\partial^4 y}{\partial x^4} = 0 \ , \qquad (1)$$

where *y=y(x,t)* transverse movement of the drill rod, m; E - the modulus of rod material elasticity, N/m^2; J - moment of inertia of the rod cross section, m^4; m - intensity rod mass, kg/m; R - rod feeding force, N.

De facto, at both ends of the rod, sliding fits, but with such a ratio of the length of the rod l and its diameter $D(l>>D)$ the rod behaves like a hinged beam, therefore the boundary conditions are written as

$$y(x,t)\big|_{x=0} = 0\, , \ \frac{\partial^2 y(x,t)}{\partial x^2}\Big|_{x=0} = 0 \qquad (2)$$

$$y(x,t)\big|_{x=l} = 0\, , \ \frac{\partial^2 y(x,t)}{\partial x^2}\Big|_{x=l} = 0 \qquad (3)$$

Initial conditions can be written as

$$y(x,t)\big|_{x=0} = \varphi(x)\, , \ \frac{\partial y(x,t)}{\partial t}\Big|_{x=0} = \psi(x)\, , \qquad (4)$$

where $\varphi(x)$, $\psi(x)$ - functions that determine the initial profile and the transverse speed of the rod.

The solution of the Cauchy problem (1),...,(4) will be made by the Fourier method [10] in the form of a product of functions depending on one variable,

$$y(x,t) = X(x)T(x)\, . \qquad (5)$$

Substituting (5) into the differential equation (1), we obtain

$$EJ\frac{d^4 X}{\partial x^4}T + R\frac{\partial^2 X}{\partial x^2} + mX\frac{d^2 T}{\partial t^2} = 0\, . \qquad (6)$$

Next, we write equations (6) in the form of relations of functions of one variable. To do this, we transfer the third term to the right side

$$EJ\frac{d^4 X}{\partial x^4}T + R\frac{\partial^2 X}{\partial x^2} = -mX\frac{d^2 T}{\partial t^2}\, ,$$

and divide both sides of the equation by the product of functions (5). After the reduction in the left side of the equality on *T*, and in the right side on *X*, we get

$$\frac{\frac{EJ}{m}\frac{d^4X}{\partial x^4}+\frac{R}{m}\frac{d^2X}{dx^2}}{X}=-\frac{\frac{d^2T}{\partial t^2}}{T} \quad (7)$$

Since the left and right side of the equation (7) depend on various variables, for equality, they must be constant, i.e.,

$$\frac{\frac{EJ}{m}\frac{d^4X}{\partial x^4}+\frac{R}{m}\frac{d^2X}{dx^2}}{X}=\omega^2, \quad (8)$$

$$-\frac{d^2T/\partial t^2}{T}=\omega^2, \quad (9)$$

where - ω - circular frequency, *rad/s.*

The equations (8) and (9) of multiplication, respectively, by *X* and *T* are reduced to the form

$$\frac{EJ}{m}\frac{d^4X}{\partial x^4}+\frac{R}{m}\frac{d^2X}{dx^2}-\omega^2X=0, \quad (10)$$

$$\frac{d^2X}{dx^2}+\omega^2T=0. \quad (11)$$

Solution of equation (10) will be sought in the form

$$X=e^{\lambda\cdot x}. \quad (12)$$

Substituting (12) into (10) we obtain the characteristic equation

$$\frac{EJ}{m}\lambda^4+\frac{R}{m}\lambda^2-\omega^2=0. \quad (13)$$

Equation (13) is biquadratic and is solved by reducing the wall thickness, length, and load to a quadratic equation.

$$\frac{EJ}{m}\xi^2+\frac{R}{m}\xi-\omega^2=0, \quad (14)$$

where $\xi=\lambda^2$

The solution of equation (14) is

$$\xi_1 = \frac{-\frac{R}{m} - \sqrt{D}}{\frac{2EJ}{m}}, \ \xi_2 = \frac{-\frac{R}{m} + \sqrt{D}}{\frac{2EJ}{m}}, \qquad (15)$$

where $D = \sqrt{\frac{R^2}{m^2} + \frac{4EJ\omega^2}{m}}$ - discriminant of the equation (14)

The solution (15) can be reduced to the form

$$\xi_1 = \frac{-R(\sqrt{1+\frac{4EJm\omega^2}{R}}+1)}{2EJ}, \xi_1 = \frac{R(\sqrt{1+\frac{4EJm\omega^2}{R^2}}-1)}{2EJ}. \qquad (16)$$

According to the change of a variable, we find four roots of bi-quadratic equations, two of which are complex-related, and two are real.

$$\lambda_1^{(1)} = \sqrt{\frac{R}{2EJ}\left(\sqrt{1+\frac{4EJm\omega^2}{R^2}}+1\right)} \cdot i,$$
$$\lambda_1^{(2)} = -\sqrt{\frac{R}{2EJ}\left(\sqrt{1+\frac{4EJm\omega^2}{R^2}}+1\right)} \cdot i, \qquad (17)$$

where ($i = \sqrt{-1}$ - imaginary unit)

$$\lambda_2^{(1)} = \sqrt{\frac{R}{2EJ}\left(\sqrt{1+\frac{4EJm\omega^2}{R^2}}-1\right)},$$
$$\lambda_2^{(2)} = -\sqrt{\frac{R}{2EJ}\left(\sqrt{1+\frac{4EJm\omega^2}{R^2}}-1\right)} \qquad (18)$$

We introduce notation to simplify further transformations, reducing the number of variables to two

$$\alpha = \sqrt{\frac{R}{2EJ}}, \ \beta = \sqrt{1+\frac{4EJm\omega^2}{R^2}}. \qquad (19)$$

Then formulas (17) and (18) take the form

$$\lambda_1^{(1)} = \alpha \cdot \sqrt{\beta+1} \cdot i, \lambda_1^{(2)} = -\alpha \cdot \sqrt{\beta+1} \cdot i, \lambda_2^{(1)} = \alpha \cdot \sqrt{\beta-1}, \lambda_2^{(2)} = -\alpha \cdot \sqrt{\beta-1}. \quad (20)$$

Taking into account (12), we obtain four linearly independent solutions

$$X_1 = e^{\lambda_1^{(1)} \cdot x}, X_2 = e^{\lambda_1^{(2)} \cdot x}, X_3 = e^{\lambda_2^{(1)} \cdot x}, X_4 = e^{\lambda_2^{(2)} \cdot x}$$

Since the original equations contain only real variables, the solution must also be expressed in terms of real variables. Considering that a linear combination of solutions is also a solution of a linear differential equation, we get

$$\overline{X_1} = \frac{X_1 + X_2}{2} = \cos\left(\alpha\sqrt{\beta+1} \cdot x\right), \overline{X}_2 = \frac{X_1 - X_2}{2i} = \sin\left(\alpha\sqrt{\beta+1} \cdot x\right), \quad (21)$$

$$\overline{X}_3 = \frac{X_3 + X_4}{2} = ch\left(\alpha\sqrt{\beta-1} \cdot x\right), \overline{X}_4 = \frac{X_3 - X_4}{2} = sh\left(\alpha\sqrt{\beta-1} \cdot x\right). \quad (22)$$

As a result, the general solution of equation (10) is written as

$$X(x) = C_1 \cos\left(\alpha\sqrt{\beta+1} \cdot x\right) + C_2 \sin\left(\alpha\sqrt{\beta+1} \cdot x\right) + \\ + C_3 ch\left(\alpha\sqrt{\beta-1} \cdot x\right) + C_4 sh\left(\alpha\sqrt{\beta-1} \cdot x\right). \quad (23)$$

The boundary conditions taking into account the representation of the function $y(x, t)$ in the form (5) will take the form

$$X(x)\Big|_{x=0} = 0, \ \left.\frac{d^2 X(x)}{dx^2}\right|_{x=0} = 0 \quad (24)$$

$$X(x)\Big|_{l=1} = 0, \ \left.\frac{d^2 X(x)}{dx^2}\right|_{l=1} = 0\,. \quad (25)$$

First of all, let's calculate the derivatives of the total solution (23)

$$\frac{dX}{dx} = (\alpha(\sqrt{\beta+1}(-C_1 \sin(\alpha\sqrt{\beta+1} \cdot x) + C_2 \cos(\alpha\sqrt{\beta+1} \cdot x)) + \\ + \sqrt{\beta-1}(C_3 sh(\alpha\sqrt{\beta-1} \cdot x) + C_4 ch(\alpha\sqrt{\beta-1} \cdot x))) \quad (26)$$

$$\frac{d^2 X}{dx^2} = \alpha^2((\sqrt{\beta+1})(-C_1 \cos(\alpha\sqrt{\beta+1} \cdot x) - C_2 \sin(\alpha\sqrt{\beta+1} \cdot x)) + \\ + (\beta-1)(C_3 ch(\alpha\sqrt{\beta-1} \cdot x) + C_4 sh(\alpha\sqrt{\beta-1} \cdot x))) \quad (27)$$

Substituting (26) and (27) into the boundary conditions (24), we obtain

$$C_1 + C_3 = 0, \ C_1(\beta+1) + C_3(\beta-1) = 0$$

Given that the determinant of a system of linear equations is not equal to zero

$$\Delta = \begin{vmatrix} 1 & 1 \\ \beta+1 & \beta-1 \end{vmatrix} = \beta - 1 - \beta - 1 = -2 \neq 0$$

This system has the only zero solution, i.e.

$$C_1 = 0\,, C_3 = 0\ . \tag{28}$$

Substituting (28) into the general solution (23), we obtain

$$X(x) = C_2 \sin(\alpha\sqrt{\beta+1}\cdot x) + C_4 sh(\alpha\sqrt{\beta-1}\cdot x) \tag{29}$$

To find other constants, we use the boundary conditions (25)

$$\begin{gathered} C_2 \sin(\alpha\sqrt{\beta+1}\cdot l) + C_4 sh(\alpha\sqrt{\beta-1}\cdot l) = 0 \\ -C_2(\beta+1)\sin(\alpha\sqrt{\beta+1}\cdot l) + C_4(\beta-1) sh(\alpha\sqrt{\beta-1}\cdot l) = 0 \end{gathered} \tag{30}$$

The resulting homogeneous system of linear equations must have non-zero solutions, since otherwise differential equation (10) will have only zero solution.

For the existence of a non-zero solution of the system of linear equations (30), it is necessary that its determinant is zero, i.e.,

$$\Delta = \begin{vmatrix} \sin(\alpha\sqrt{\beta+1}\cdot l) & sh(\alpha\sqrt{\beta-1}\cdot l) \\ -(\beta+1)\sin(\alpha\sqrt{\beta+1}\cdot l) & (\beta-1)\cdot sh(\alpha\sqrt{\beta-1}\cdot l) \end{vmatrix} = 0 \tag{31}$$

Expanding the determinant, we obtain the transcendental equation

$$\begin{gathered} (\beta-1)\sin(\alpha\sqrt{\beta+1}\cdot l) sh(\alpha\sqrt{\beta-1}\cdot l) + \\ + (\beta+1)\sin(\alpha\sqrt{\beta+1}\cdot l) sh(\alpha\sqrt{\beta-1}\cdot l) = 0 \end{gathered}$$

or, after algebraic transformations,

$$2\beta \sin(\alpha\sqrt{\beta-1}\cdot l) sh(\alpha\sqrt{\beta-1}\cdot l) = 0\,. \tag{32}$$

In the future, for the convenience of calculations we introduce substitute

$$\gamma = \alpha \cdot l\,. \tag{33}$$

Taking into account (33), the equation (32) with the condition β>0 takes the form

$$\sin(\gamma\sqrt{\beta+1) = 0}\,. \tag{34}$$

Equation (34) admits the solution at values

$$\gamma\sqrt{\beta+1} = \pi \cdot n, \ \ (n = 1,2,...), \tag{35}$$

From equation (35) we find

$$\beta_n = \frac{\pi^2 n^2}{\gamma^2} - 1, \ (n = 1,2,...), \tag{36}$$

Then, we consider the substitute (19), we find the discrete values of the circular part ω

$$\omega_n = \frac{R}{2}\sqrt{\frac{\beta_n^2 - 1}{EJm}}, \ (n = 1,2,...), \tag{37}$$

We take into account (36), formula (37) is consistently converted into

$$\omega_n = \frac{R}{2\sqrt{EJm}}\sqrt{\left(\frac{\pi^2 n^2}{\gamma^2} - 1\right)^2 - 1},$$

$$\omega_n = \frac{R}{2\sqrt{EJm}}\frac{\pi \cdot n}{\gamma}\sqrt{\frac{\pi^2 n^2}{\gamma^2} - 2}, \tag{38}$$

$$\omega_n = \frac{\pi^2 n^2}{l^2}\sqrt{\frac{EJ}{m}}\sqrt{1 - \frac{2\gamma^2}{n^2\pi^2}}, \quad (n = 1,2,...).$$

Cyclic frequency is according to the formula

$$f_n = \frac{\omega_n}{2\pi}, \ (n = 1,2,...). \tag{39}$$

Considering that the value (35) nullifies the determinant (31) of system (30), to find the constants we have one equation with two unknowns

$$C_2 \sin \pi \cdot n + C_4 sh(\gamma\sqrt{\beta_n - 1}) = 0, \tag{40}$$

or, taking into account (32) i (36),

$$C_4 sh(\lambda\sqrt{\left(\pi^2 n^2\right)/\gamma^2 - 2}) = 0, \text{ i.e. } C_4 = 0.$$

As a result of the solution, equation (29) takes the form

$$X_n(x) = C_2 \sin\left(\pi \cdot n \frac{x}{l}\right), \ (n = 1,2,...), \tag{41}$$

At the same time, circular frequencies (38) are eigenfrequencies, and functions (41) are eigenfunctions that correspond to these frequencies. The time dependence of the solution of the differential equation (1) is found by solving equation (11). To solve this equation, we compose the characteristic equation $k_2 + \omega^2 = 0$, the solution of which has the form

$$k_{1,2} = \pm i \cdot \omega . \tag{42}$$

Then, taking into account (42), the solution of equation (11) is written in the form

$$T(t) = A\cos\omega \cdot t + B\sin\omega \cdot t . \tag{43}$$

Then, taking into account (38), the solution of equation (43) is written in the form

$$T_n(t) = A_n \cos\omega_n t + B\sin\omega_n t, \ (n = 1,2,...), \tag{44}$$

is written in the form $y_n(x,t) = X_n(x)T_n(t), \ (n = 1,2,...),$ or, taking into account (41) i (44),

$$, y_n(x,t) = \sin\left(\pi \cdot n\frac{x}{l}\right)\left(c_n\cos\omega_n t + d_n\sin\omega_n t\right), \tag{45}$$

where $c_n = C_2 a_n$, $d_n = C_2 b_n$, Then the solution of the differential equation (1) can be represented as a superposition of solutions (45)

$$y(x,t) = \sum_{n=1}^{\infty}\sin\left(\pi \cdot n\frac{x}{l}\right)\left(c_n\cos\omega_n t + d_n\sin\omega_n t\right). \tag{46}$$

To find the constants included in the solution (46), it is necessary to use the initial conditions (4). According to the first initial condition, we get

$$\varphi(x) = \sum_{n=1}^{\infty} c_n \sin\left(\pi \cdot n\frac{x}{l}\right). \tag{47}$$

To find the constant c_n, multiply both sides of equality (47) by $\sin(\pi \cdot n(kx/l))$and integrate over the interval [0, l], that gives

$$\int_0^1 \varphi(x)\sin\left(\pi k\frac{x}{l}\right)dx = \frac{l}{2}\sum_{n=1}^{\infty} c_n\delta_{kn} = \frac{l}{2}c_k , \tag{48}$$

where $\delta_{kn} = \begin{cases} 0, k \neq n \\ 1, k = n \end{cases}$ - Kronecker symbol. Thus, according to (48) we can write

$$c_n = \frac{2}{l}\int_0^1 \varphi(x)\sin\left(\pi \cdot n\frac{x}{l}\right)dx. \quad (49)$$

To find the second constant, we first pre-differentiate the solution (46) with respect to time

$$\frac{\partial y(x,t)}{\partial t} = \sum_{n=1}^{\infty}\sin\left(\pi \cdot n\frac{x}{l}\right)(-\omega_n c_n \sin\omega_n t + \omega_n d_n \cos\omega_n t). \quad (50)$$

Then, according to the second initial condition, we get

$$\psi(x) = \sum_{n=1}^{\infty}\omega_n d_n \sin\left(\pi \cdot n\frac{x}{l}\right). \quad (51)$$

After multiplying both sides of equation (51) by sin (π·*n*(*kx/l*)) and integrating on the interval [0, l], we obtain

$$d_n = \frac{2}{\omega_n l}\int_0^1 \psi(x)\sin\left(\pi \cdot n\frac{x}{l}\right)dx. \quad (52)$$

The solution of the Cauchy problem (1), ... (4) taken into account (49) and (52) is written as

$$y(x,t) = \frac{2}{l}\sum_{n=1}^{\infty}\sin\left(\pi \cdot n\frac{x}{l}\right)(\cos\omega_n t\int_0^1 \varphi(\xi)\sin\left(\pi n\frac{\xi}{l}\right)d\xi + \quad (53)$$

$$+\frac{1}{\omega_n}\sin\omega_n t\int_0^1 \psi(\xi)\sin\left(\pi n\frac{\xi}{l}\right)d\xi).$$

To simplify research, we will bring formula (38) to a dimensionless form.

$$\tilde{\omega}_n = n^2\sqrt{1-\frac{\delta^2}{n^2}},\ \ (n = 1,2,...), \quad (54)$$

where $\tilde{\omega}_n = \frac{\omega_n}{\overline{\omega}}$, $\overline{\omega} = \frac{\pi^2}{l^2}\sqrt{\frac{EJ}{m}}$, $\delta = \frac{l}{\pi}\sqrt{\frac{R}{EJ}}$

In formula (54), the unit of measurement of the circular frequency is $\overline{\omega}$, which is determined by the properties of the studied drill rod, and the effect of the load on the rod is determined by the complex δ. Such a record of formula (54) allows one to study the effect of parameters not separately, but in a complex, reducing their number from five (*R*,*E*,*J*,*m*,*l*) to two $(\tilde{\omega}, \delta)$.

Table 1 presents the results of calculations by the formula (54). Analysis of the results calculation shows that for the first harmonic (n=1), with increasing parameter δ, the circular frequency sharply decreases.

For further harmonics (n=2.3), there is no such a change - the magnitudes of the circular frequencies are larger, but they vary considerably less depending on the parameter δ Therefore, we can conclude that the first harmonic makes the main contribution to the vibration of the drill rod.

Table 1

The dependence of the angular frequency of the load for different harmonics in a non-dimensional form

δ \ n n	1	2	3
0,2	0,98	3,98	8,98
0,4	0,92	3,92	8,92
0,6	0,8	3,82	8,82
0,8	0,6	3,67	8,67
1	0	3,46	8,49

Below are the results of numerical simulation. According to the initial data, the parameters of the drill rod are characterized by

$$R = 200\,kN;\, m = 206{,}3;\, E = 2 \cdot 10^{11}\,Pa;\, J = 9{,}72 \cdot 10^{-5}\,m^4;$$

$$d_1 = 9{,}112\,m;\, d_2 = 0{,}215\,m.$$

Then, according to the formula (54), we find, rad/s

$$\delta = \frac{l}{\pi}\sqrt{\frac{R}{EJ}} = \frac{l}{3{,}14159}\sqrt{\frac{200000}{2 \cdot 10^{11} \cdot 9{,}72 \cdot 10^{-5}}} = 0{,}032 \cdot l,$$

$$\overline{\omega} = \frac{\pi^2}{l^2}\sqrt{\frac{EJ}{m}} = \frac{3{,}14^2}{16^2}\sqrt{\frac{2 \cdot 10^{11} \cdot 9{,}72 \cdot 10^{-5}}{206{,}3}} = \frac{3{,}03 \cdot 10^3}{l^2},$$

$$\tilde{\omega}_n = n^2 \cdot \sqrt{1 - \frac{\delta^2}{n}} = \sqrt{1 - 1{,}042 \cdot 10^{-3} l^2}.$$

Formulas for circular and cyclic frequencies take the form

$$\omega_n = \overline{\omega} \cdot \tilde{\omega}_n = 3{,}03 \cdot 10^3 \cdot \frac{n^2}{l^2}\sqrt{1 - 1{,}042 \cdot 10^{-3}\frac{l^2}{n^2}};$$
$$f_n = 482{,}239 \cdot \frac{n^2}{l^2}\sqrt{1 - 1{,}042 \cdot 10^{-3}\frac{l^2}{n^2}}. \quad (n = 1,2,...). \tag{55}$$

Table 2 presents the results of calculations according to the formula (55) of circular and cyclical natural frequencies of transverse oscillations of the drill rod of the first three harmonics for different lengths of the drill rod.

Table 2. The calculation results for the three harmonics of the natural frequencies of oscillations for different lengths of the drill rod.

l, ì	8			16		
n	1	2	3	1	2	3
ω_n, rad/s	45,74	187,79	424,51	11,37	45,74	104,93
f_n, Hz	7,28	29,89	67,56	1,81	7,28	16,70

Checking the adequacy of the mathematical model was performed using the SolidWorks software package [11-15].

On the linear dimensions of two heavy drill rods, taking into account the requirements of the COSMOSWorks program [13], a computer model of the drilling rod was built: material - steel 45; connection of two rods - threaded coupling; axial force P = 220 kN; end fixing - the upper part in the form of a sliding fit of the spindle sleeve in the support node, the lower part - a dril bit in the form of a ball five.

The specified operating (excitation) frequency range is from 0 to 106.8 rad/s (0-17 Hz); according to the COSMOSWorks program, the modal damping factor was chosen, corresponding to the “metal structure with connections” - 0.03. The calculation of the amplitudes of oscillations in a given frequency range was carried out every 0.2 Hz, and near the resonant frequencies - every 0.05-0.1 Hz. Further, using SolidWorks, and FFiplus applications are performed calculations presents table 3.

Table 3

List of modes of rotation of the drill rod.

Mode №	Frequency (Rad/sec)	Frequency (Hertz)	Period (Seconds)
1 – x-axis	11.842	1.8847	0.5306
1 – y-axis	11.843	1.8849	0.53054
2 – x-axis	47.296	7.5274	0.13285
2 – y-axis	47.319	7.5311	0.13278
3 – x-axis	106.17	16.897	0.059183
3 – y-axis	106.29	16.916	0.059114

The visualization of the oscillation amplitudes of the drilling rod by the COSMOSWorks program is performed in a stylized way. The half-cycles of vibration amplitudes are shown on one side of the axis of rotation of the drilling rod. This indicates their symmetry. In the first mode, one half-period is the length of the drilling rod, in the second mode - one full period, in the third mode - 1.5 periods of oscillation.

The theoretical calculations in the first mode differ from the computer experiment in circular frequency by 0.75% and in amplitude by 0.04%, which can be considered a good confirmation of the theoretical calculations.

The obtained calculation results are in good agreement with both experimental data and modeling performed in the SolidWorks environment.

The findings, the task of further research.

Mathematical modeling of the transverse oscillations of the drill rod, based on physical patterns, allowed us to establish the functional dependence of the natural frequencies of the transverse oscillations of the drill rod on the main parameters of the rod: mass intensity, elastic modulus, diameter, wall thickness, length and axial load.

Bringing the formula that determines the frequency dependences of the bar parameters to a dimensionless view allowed grouping these parameters into complexes, reducing the number of variables from five to two, and thereby greatly simplifying the study of the resulting dependence.

Analysis of the dependence of the drill rod transverse oscillation frequency on the load showed that the first harmonic plays a significant role in the transverse oscillations. The frequencies of the following harmonics are much higher, but depend little on the load.

The task of further research is to implement mathematical modeling of the amplitudes of the forced transverse oscillations of the drill rod and, due to scale invariance or scaling, will allow, based on the similarity theory and dimension analysis, to study the effect of parameters not separately from the amplitudes of the forced transverse oscillations but in the complex, which significantly reduce the total amount of research.

References

1. **Marasanov Yu.P., Borovkov V.A., Stromwasser P.C.** (1972). *Elastic oscillations of a drill rod when drilling directional wells with a roller bit.* Moscow: News of higher educational institutions. Mining Journal. P. 132–140.

2. **Gromadskyi A.S., Gromadskyi V.A., Aksenov A.V.** (2011, May 25–28). *Damping of longitudinal oscillations of the rotary and drilling rod of rotary drilling rigs.* Kryvy Rih: Sustainable development of the mining and metal industry. International Sci.-Tech. Conf. P. 133

3. **Saroyan A.Ye**. (1979). *Drillstrings in deep drilling* Moscow: Nedra.

4. **Beshta A.S., Khilov V.S., Plakhotnik V.V**. (2004). *Mathematical model of longitudinal and torsional oscillations of a drilling rig.* Mining mechanics and automation. Collection of scient**ific papers. Issue 73.**

5. Simonov V.V., Yunin E.K. (1977). *The influence of oscillatory processes on the operation of the drilling tool*/Moscow: Nedra, 212 p.

6. **Kantovich L.I., Babarika S.D**. (1986*). Comprehensive assessment of the effectiveness of the stabilization of the drilling rod at rotary drilling.* Mining Journal. - № 2. - P. 50–51.

7. **Sukhanov A.F., Kutuzov B.N., Schmidt R.G**. (1969). *Vibration and reliability of rotary drilling rigs operation.* Moscow: Nedra. 123 p.

8**. Khilov V.S., Plakhotnik V.V**. (2004). *Evaluation of the natural frequencies of oscillations of the drill rod in non-stationary modes.* Collection of scientific papers of the National Mining University. –№ 19, v. 4. - p. 145–150.

9. **Timoshenko S.P., Young D.H., Weaver U.** (1985) *Variations in engineering*. Moscow: Mechanical engineering – 472p.

10. **Beshta A.S., Khilov V.S., Plakhotnik V.V. (**2004). *A mathematical model of the longitudinal and torsional vibrations of the drill rod.* Mining mechanics and automation. - Collection of scientific papers. - Issue 73.

11. *Zhupiev A.L., Bezpalko T.V., Zinoviev S.N*. (2005). *Methods of using the package of SolidWorks Education in the process of training of mechanical engineers.* Science Bulletin of the National Minig University - № 11. - P. 26–31.

12. **Kuiva Lee. (**2004). Basics of CAD/CAM /CAE. - S.-Pb .: Petersburg. 560 p.

13. Company SolidWorks Russia. The composition and purpose of the program complex of educational process SolidWorks Education. http://www.solidworks.ru/swr-academy/.

14**. Dunaeva N.Yu., Zagayko S.A.** (2007). SolidWorks-2007. S.Peterburg. S.Pb .: BHV - Petersburg, 1328 p.

15. **Alyamovsky A.A**. SolidWorks-2007. *Computer modeling in engineering practice*. S.Pb .: BHV - Petersburg, 2007. - 1040 p.