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A competency-based approach to the systematization of mathematical problems in a specialized school

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Abstract. The issue of searching for new methodological approaches to the systematization that will encourage the increase of students motivation to learn mathematics under the competency-based approach is considered in this article. The research analyzes the existing works on the increase in students motivation to learn Mathematics, in particular, the use of cross-curricular connections while forming students competency. Competency-based problems, systematized according to the topic of the 10th grade Functions, their features and graphics, were determined as the tools to measure students competency and a method to form their motivation to learn mathematics. The use of such methods as the initial research and information gathering, systematization and structural analysis of the problems, data processing allowed the authors of the article to systematize the problems for school subjects of the 10th grade that demonstrate cross-curricular connections of Mathematics with other learning subjects and allow showing the advantages of the mathematical modeling in researching real processes. The research shows the realization of cross-curricular connections in time and such connections as parallel learning, perspective connections, use of the mathematical modeling method are shown. An experiment was held in order to prove the efficiency of implementing a system of the problems to demonstrate the use of the function in different tasks of natural subjects. The results proved that the implemented system of problems considerably influences the increase in students motivation to learn mathematics.

1. Introduction

According to the methodological recommendations on developing the components of the state standard for basic and complete general secondary education based on the competency-based approach, the objective to form, develop and improve the complex of the students competencies during the education arises for the system of secondary education. For subject teachers, the transfer to the competency-based approach means improving the system of problems to increase students motivation according to the requirements of learning programs for students of general secondary schools. Mathematics is not an exception, that following the competency-based approach, also implies the research of new methodological approaches and systematization of problems that will encourage the increase of students motivation to learn mathematics. That



why learning Mathematics should contribute to forming students' key competencies among which there are mathematics competency and main competencies of natural sciences and technologies. One of such approaches is the demonstration of a constant connection between Mathematics and other natural sciences to the students. In order to ensure it, the learning programs include the determination of cross-cutting lines of the key competencies: Environmental security and traditional development, Civil responsibility, Health and security, Entrepreneurship and financial literacy that are focused on forming an ability to use knowledge and skills in real-life situations among students. Thus, while learning Mathematics a need to select and systematize the problems that allow demonstrating the use of Mathematics in other subjects to the students, which will increase their motivation to learn Mathematics, becomes quite urgent.

2. Literature review

The introduction of Mathematics to the compulsory subjects of external testing did not solve the problem of improving students motivation to learn mathematics. The question of searching for ways and approaches to overcome students fear and their difficulties during learning this subject has always interested scientists (Posamentier [1], Abramovich, Grinshpan and Milligan [2], Hernandez-Martinez and Vos [3], Langoban and Tan [4], Williams and Williams [5], Cody [6], Vlasenko et al. [7, 8], and others). For instance, Posamentier [1] distinguishes external and internal motivations. In the internal students motivation during learning Mathematics the scientist sees students understanding why they need to learn mathematics, where this knowledge can be used. The scientist also determines nine methods of improving students motivation to learn mathematics, one of which is the demonstration of the benefits to learn mathematical terms. The researchers Abramovich, Grinshpan, Milligan [2], Lovianova et al. [9] emphasize that Mathematics significantly developed and entered all life areas. Considering Mathematics as a necessary subject, scientists indicate the power of mathematical modeling that should serve as motivation to develop students competencies [10, 11]. Hernandez-Martinez and Vos [3] agree with this idea and say that students motivation can be increased through the demonstration of using Mathematics in the real world. Langoban and Tan [4], Vlasenko et al. [12] believe that the efficiency of teaching Mathematics will increase if students are motivated through the demonstration of the efficiency of using Mathematics in practice. K. C. Williams and C. C. Williams [5] define five components of students motivation to learn mathematics: a student, a teacher, content, method, and environment, and emphasize that mathematical problem have a particular learning content and allow forming and developing internal motivation of students learning activity. Cody [6] sees the increase of students motivation to learn Mathematics through learning activity that is ensured while solving mathematical problems systematized according to a particular approach.

Aimed at increasing students motivation to learn mathematics, we chose an approach the implementation of which requires: 1) arrangement of learning programs in natural subjects in order to enable giving learning material where the use of mathematical concepts is described in parallel; 2) the realization of perspective cross-curricular connections of Mathematics with other learning subjects on condition that it is impossible to provide the material in parallel. The mentioned approach will encourage students understanding of how to use the acquired knowledge in Mathematics in other subjects ensuring both the formation of mathematics modeling skill and increase of student's motivation to learn. While choosing an approach it was also considered that the motivation is internal if it corresponds to the aim of the students activity. The conditions of learning activity, under which learning the content of the subject will be both the motivation and aim, are ensured. It is taken into account that internal motivations are connected with the students cognitive need, their satisfaction that is obtained during the learning period, and as a result, the dominance of internal motivation is characterized by the display of students involvement in the learning activity.

In order to ensure this approach several problems should be systematized. According to Ghanbari [13], Feng, Lu, and Yao [14], Jacome [15] competency-based problems have the greatest potential in this area. In this case, the problems that simulate the actual problems that arise while learning other subjects become the subject of the students learning activity. Niss and Bruder [16], Cai and Hwang [17], Vlasenko et al. [18] propose to consider competency-based problems as specially designed tasks aimed at forming a dynamic combination of knowledge and practical skills, ways of thinking, professional and philosophical qualities that allow to successfully carry out further educational and professional activities. Scientists believe that the tasks of the scientific type can be related to such types of problems.

Considering that one of the main content areas of the course Mathematics in high school is the functional area, the article is aimed at the systematization of competency-based problems following the topic of the 10th grade Functions, their features and graphics. Also, the increase in students motivation to learn Mathematics through the demonstration of the tight connection between Mathematics and other learning subjects is proven.

3. Methods

While choosing the problems that will be included in the system, it was considered that the key message of the topic Functions, their features and graphics should be the modeling of real processes using functions. Since working with diagrams, pictures, graphics is one of the spread types of a persons practical activity, the main objectives of learning a topic include the development of students graphic culture. First of all, it is about reading graphics, in other words, setting the features of some function using its graphics. So, while choosing the system of problems for the 10th grade aimed at revealing cross-curricular connections on the example of the topic: Functions, their features ad graphics with other subjects, the following methods were used:

- primary research and data gathering: analysis of textbooks on different subjects of the 10th grade [19–27]; analysis of mathematics textbooks of the 10th grade [28–32];
- systematization and structural analysis: selected problems on different subjects, the priority was given to competency-based problems;
- data processing: the essence of mathematical modeling to show the use of functions in problems for different learning subjects of the school course was used.

Based on such research methods, the problems for learning subjects of the 10th grade were systematized: physics, economics, chemistry, biology, information technologies, art, environment, geography, civil education, law, homeland defense, history of Ukraine that require the use of students knowledge on the topic Functions, their features and graphics. Lets look at the examples of the problems concerning the developed system (table 1).

Table 1. Problems where students use their knowledge on the topic Functions, their features and graphics.

Subject Topic	Problem
Economics. Market structure: characteristics of different markets. Capital market. Buyers and sellers of credit resources. What influences demand and offer in the capital market	It is known that the demand for bricklayers is described by the function $LD = 160.06W$, where W is a day salary. One bricklayer is ready to work for no less than 200 UAH/day; two bricklayers are ready to start working for no less than 150 UAH, three bricklayers – no less than 100 UAH, and two others – no less than 120 UAH. Build the demand and offer curve in the bricklayers market. Characterize the balance and its change if housing construction is getting active [19, p. 181].
Biology. Biodiversity. Prokaryotic organism: bacteria	Bacteria reproduction. Consider the situation when one prokaryotic cell is found in optimal conditions and reproduce without any obstacles. How many cells will be in a model population in ten generations if the cell division takes place every 20 min? Fill in the table and build the graphics of population growth (at the horizontal axis – time, at the vertical axis – cell number). Write the mathematical equation that reflects the pattern of population growth [20, p. 44].

0 20 40 60 80 100 120 140

Time, min
Number of
bacteria

Environment. Technogenesis and economics. Technogenesis – technical progress, economic growth

The graphics of structure change in consuming fuel and energy resources in the world in the 20th century are represented in the figure 1. Make a conclusion [21, p. 56].

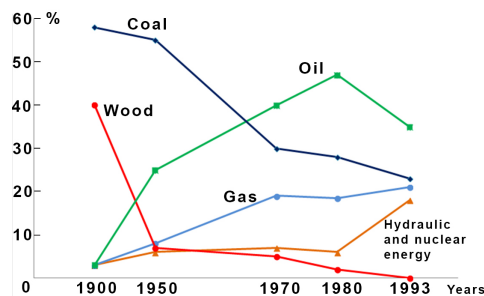


Figure 1. The graphics of structure change in consuming fuel and energy resources in the world in the XXth century.

Physics. Mechanics. Straight equally accelerated motion

According to the graphics in the figure 2, write the dependency equation $v_x = v_x(t)$ $x = x(t)$. Consider that at the initial stage ($t = 0$) the body is at the beginning of the coordinates ($x = 0$). Build the dependency graphics $x = x(t)$ for every body [22, p. 22].

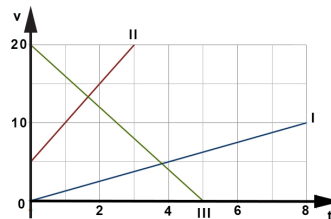


Figure 2. Body motion graphs.

Art. Art trip around European countries. Italian art: the creative boom of humanity

The tower of Pisa is one of the main remarkable architectural monuments of Italy. The external part of every gallery of the tower creates the columns with classical chapiters that lean on the closed arches. The top of the tower has a belfry. The slope of the tower is approximately 10%. Before the repair work, it deviated to the south by 5.5 degrees, now it is deviating by 4 degrees. It is reflected in numbers in the following way: the arch cornice is shifted by 4.5 meters in comparison to the lower one. What function – increasing or decreasing – do you have an association with that is connected with this architectural monument?

Geography. General characteristics of Europe. Natural conditions of Europe

Compare the climate of the cities Reykjavik (Iceland), Rome (Italy), Arkhangelsk (Russia), Kassel (Germany) using climatograms (see the figure 3). Explain the differences [23, pp. 43-44].

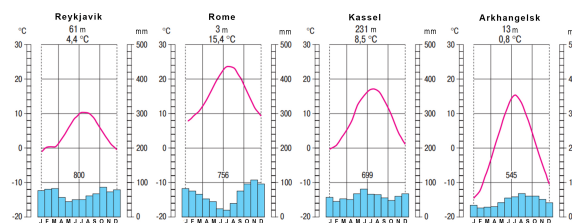


Figure 3. The climate of the cities Reykjavik (Iceland), Rome (Italy), Arkhangelsk (Russia), Kassel (Germany).

Civil education. Interaction between citizens and the state in order to achieve public welfare and offer, market price, competition

Using the table, show the curve of offer and demand. Determine the balanced price for bicycle, and also its surplus and deficit. Fill in the blank cells of the table [24, p. 188].

Price, UAH	Demand, item	Offer, item	Surplus of the offer, comparison to the demand, item	Type of the market situation: (S – surplus, D – deficit, E – equality)
3500	0	130		
3000	12	122		
2400	27	108		
2000	38	97		
1650	50	85		
1300	66	70		
1150	60	60		
1000	88	56		
750	120	42		

Chemistry. Carbohydrate. Alkanes: physical and chemical characteristics.

Analyze the graphics in the figure 4. According to the melting and boiling temperature, determine the intervals where the data for gas, liquid, and solid alkanes are given. Melting temperature (line A), boiling temperature (line B) of normal alkanes [25, p. 40, 47].

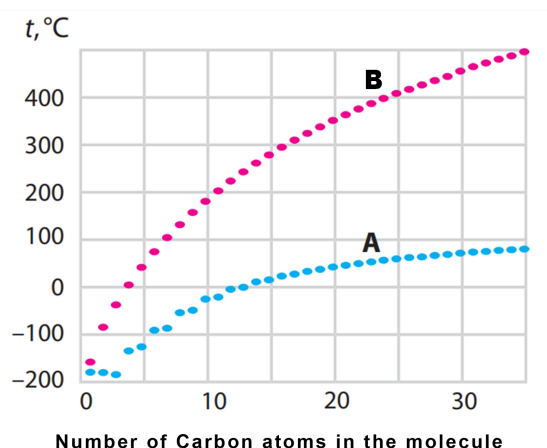


Figure 4. Analysis graphs.

Information technologies. Models and modeling. Data analysis and visualization. Visualization of data trends and ranges. Infographics.

The table represents the number of registered flu and SARS cases for every 10 thousand people in city H:

No. of weeks	1	2	3	4	5	6
Number of people who fell ill for 10 thousand people	150	130	145	120	125	110

Based on the table data build a dotted diagram and make a forecast: In how many weeks the epidemiological barrier of flu and SARS cases, which is 50 cases for 10 thousand people, will be exceeded in the city [26, p. 77].

History of Ukraine. World War I. The end and consequences of the war

Look at the graphics (figure 5, figure 6). What conclusions can you make? [27, p. 44].

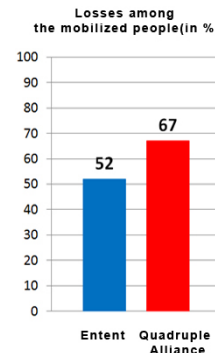
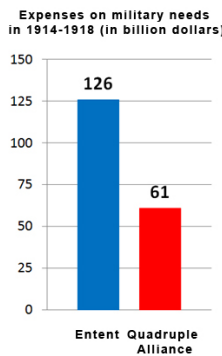


Figure 5. Expenses on military needs in 1914-1918 (in billion dollars).

Figure 6. Losses among the mobilized people (in %).

Law. Structure of the state. A nations form of government.

Study the number of European countries with monarchy and republic forms of government since 1800 and till modern times every 20 years. Fill in the table and build the graphic for every form of government, compare the graphics.

Homeland defense. Applied physical training. Strength training and overcoming obstacle

Preparing for the route march during one month of every day, running 3 km, mark the time in the table and build a graphic of the dependency between the number of training days and time that is spent on the distance. Make a conclusion.

Besides, the problems from Mathematics [28, 30, 32], algebra, and basics of calculus [29, 31] textbooks of the 10th grade on the topic Functions, their features, and graphics were systematized and they show cross-curricular connections between Mathematics and other learning subjects and enable us to show the advantages of mathematical modeling in the research of real processes (table 2). The focus was on the mathematical problems, the content of which is connected with the above-mentioned subjects such as Physics, Economics, Chemistry, Biology, Informatics, Art, Environment, Geography, Civil education, Law, Homeland defense, History of Ukraine. It was considered that the demonstration of cross-curricular problems during mathematics lessons allows increasing their motivation to learn Mathematics as a versatile language to research the processes known to students while learning other subjects.

Table 2. Cross-curricular connections of Mathematics with other learning subjects.

Mathematical problem	The subject where there is a connection																					
A skydiver jumps from the plane that is moving at the height of 1020 m above the ground. His way h (in meters), which takes place under condition of free fall, is determined using the formula $h = \frac{gt^2}{2}$, where t is time (in seconds), g is a criterion, $g \approx 10 \text{ m/s}^2$. The parachute opened at the height of 975 m above the ground. Use a formula to show the dependency of the skydivers free fall time from the mileage, build the graphic of the defined dependency, and find out how much time the skydiver flew during the free fall.	Physics																					
The Sun weight is approximately $2 \cdot 10^{27}$ t, and the Neptune weight is approximately $1 \cdot 10^{23}$ t. Use this data and also find the data about the weight of other planets and show graphically the dependency of the weight from the planet size in increasing order.	Astronomy																					
The amount of electricity consumption by a family in the first 6 months of 2012 and 2017 is represented in the table (in kw-h). Build the graphics of electricity consumption in 2012 and 2017 in one coordinate system. Make conclusions.	Economics																					
<table style="width: 100%; border-collapse: collapse; margin: 0 auto;"> <thead> <tr> <th style="width: 5%;"></th> <th style="width: 10%;">1</th> <th style="width: 10%;">2</th> <th style="width: 10%;">3</th> <th style="width: 10%;">4</th> <th style="width: 10%;">5</th> <th style="width: 10%;">6</th> </tr> </thead> <tbody> <tr> <td style="border-top: 1px solid black;">2012</td> <td style="border-top: 1px solid black;">150</td> <td style="border-top: 1px solid black;">130</td> <td style="border-top: 1px solid black;">145</td> <td style="border-top: 1px solid black;">120</td> <td style="border-top: 1px solid black;">125</td> <td style="border-top: 1px solid black;">110</td> </tr> <tr> <td style="border-bottom: 1px solid black;">2017</td> <td style="border-bottom: 1px solid black;">105</td> <td style="border-bottom: 1px solid black;">100</td> <td style="border-bottom: 1px solid black;">90</td> <td style="border-bottom: 1px solid black;">90</td> <td style="border-bottom: 1px solid black;">95</td> <td style="border-bottom: 1px solid black;">70</td> </tr> </tbody> </table>		1	2	3	4	5	6	2012	150	130	145	120	125	110	2017	105	100	90	90	95	70	
	1	2	3	4	5	6																
2012	150	130	145	120	125	110																
2017	105	100	90	90	95	70																

The table shows the dependency of potassium bromide solubility from the solvent temperature Chemistry

t°	0	20	40	60	80	100
m	55.2	65.1	75.1	85.3	95.2	104.9

Build a dependency graphic and find out if this dependency, at least approximately, follows the linear law.

Paramedics found that a child aged a years old $a \leq 18$ for normal development has to sleep during t hours per 24, where t is determined by the formula $t = 160.5a$. Find $t(16)$, $t(15)$, $t(14)$. Biology

Lets suppose that in some big pond the number of water lilies doubles every day. If at the beginning there were five water lilies, how many of them will there be in 1, 2, 3, 5, 10 days? Give a general formula for the number A_n of water lilies in n days. How many would there be in 30, 60 days, if the pond were quite big? Draw a graphic scheme of the function $n \rightarrow A_n$. Environment

It is considered that while diving under every 30.5 m the inside temperature of the Earth increases to $1^\circ C$. At the depth of 5 m, it is $15^\circ C$. Show the dependency of the temperature t from the depth h . What is the temperature at the depth of 1 km, 3 km? Geography

4. Results

We conducted an experiment in order to determine the efficiency of implementing a system of problems that show the use of functions in different tasks of natural subjects. A survey of learning motivation focus by T. D. Dubovitska was chosen to detect the motivation of the learning activity among students [33]. The methodology detects the focus and motivation level of the learning activity while learning mathematics.

The research basis included secondary schools where master students of the specialization 014 Secondary education (Mathematics) had their pedagogical practice: Kryvyi Rih educational complex No. 81, Kostiantynivka comprehensive school of the I-III levels No. 1, Kostiantynivka educational complex Comprehensive school of the I-III levels – preschool, Kramatorsk educational complex Comprehensive school of the I-III levels No. 6, Kryvyi Rih comprehensive school No. 75 and 122, Piatykhatky comprehensive school No. 3, Zelenodolsk comprehensive school No. 2. Master students who took part in the experiment attended training sessions to get acquainted with its aim and objectives and participated in the development of the system of problems and methods of conducting classes using this system. The teachers were told about the experiment and helped students to carry it out.

The main objectives of the experiment were:

- Analysis of school subjects that are learned in the 10th grade;
- Research of the learning process in algebra and basics of analysis in the 10th grade;
- Development and implementation of the system of competency-based problems on the topic Functions, their features and graphics that show the connection with other subjects;
- carrying out the analysis of the experiment results.

Control (CG) and experimental group (EG) were formed according to the methodology of detecting the focus of learning motivation offered by T. D. Dubovistka [33]. At the beginning and after the end of the experiment, a survey among the students of the 10th grade was held in order to determine their level of motivation to learn Mathematics (table 2). The students read every statement and expressed their attitude to Mathematics using the offered marks: true: +, possibly true: +, possibly false: , false: .

The experiment was held during three weeks (the time spent on the topics Functions, their features and graphics), where 163 students took part: 81 in CG and 82 in EG:

- the control group (CG) included the students of the following schools: Kryvyi Rih educational complex No. 81, Kostiantynivka comprehensive school of the I-III levels No. 1, Kryvyi Rih comprehensive school No. 75, Zelenodolsk comprehensive school No. 2, where teaching algebra is provided using the traditional methodology;
- the experimental group (EG) included the students of the following schools: Kostiantynivka educational complex Comprehensive school of the I-III levels – preschool, Kramatorsk educational complex Comprehensive school of the I-III levels No. 6, Kryvyi Rih comprehensive school No. 122, Piatykhatky comprehensive school No. 3, where teaching algebra was provided using the methodology of demonstrating cross-curricular connections of Mathematics through a system of problems on the topic Functions.

Table 3. The results of the survey among the students of control and experimental groups.

Statement	Number of points			
	CG		EG	
	Before	After	Before	After
1. Learning Mathematics allows me to learn a lot of important information for me, to show the skills	40	55	37	72
2. Mathematics is interesting for me and I want to now as much as possible about this subject	30	51	35	63
3. The knowledge I get during the lessons is enough for me while learning Mathematics	64	60	80	62
4. Mathematics lessons are not interesting for me, I do tasks because the teacher requires it	62	45	63	33
5. Difficulties that arise while learning Mathematics make it even more exciting for me	32	53	29	58
6. I read additional literature except for textbooks and references independently while learning Mathematics	24	43	27	69
7. I reckon that we could skip difficult theoretical questions while learning Mathematics	18	27	20	57

8. If something goes wrong in mathematics, I try to work it our and find a solution	49	61	47	75
9. During mathematics lessons Ive got such a feeling that I dont want to study anything	72	50	78	48
10.I work actively and do the tasks only under the teachers control.	19	36	21	54
11.I discuss with interest the material that is learned during mathematics lessons when I have free time (during the break or at home) with my classmates (friends)	27	43	26	63
12.I try to do the tasks on Mathematics by myself, I dont like when I am given a hint or help	47	50	44	71
13.If possible I try to copy classmates tasks or ask somebody to do the task instead of me	27	23	56	27
14.I believe that all the knowledge in mathematics is valuable and if possible it is necessary to know as much as possible	31	53	33	62
15.The mark I get on Mathematics is more important for me than the knowledge	20	16	55	19
16.If I am badly prepared for the lesson, I am not particularly upset and dont worry	54	45	61	37
17.My interests and hobbies in free time are connected with mathematics	19	22	17	68
18.Mathematics is difficult for me and I have to make myself do the tasks	42	32	47	17
19.If I miss the mathematics lessons due to some illness (or any other reason), I am upset	42	48	41	56
20.If it could be possible, I would exclude Mathematics from the curriculum (the learning plan)	70	63	69	54

Note that the statements in the survey show positive (1, 2, 5, 6, 7, 8, 10, 11, 12, 14, 17, 19) and negative (3, 4, 9, 13, 15, 16, 18, 20) motivation to learn Mathematics. Analyzing the obtained data, we observe that at the beginning of the experiment in both control and experimental groups on such parameters of motivation assessment as 7, 10, 17, 6, 11 the number of points scored does not exceed 30. Data on the same parameters after experimental training showed that in the experimental group the number of points on the studied parameters was in the range from 54 to 69. While in the control group only on parameters 6, 10, 11 the score was in the range from 32 to 43 points, on other parameters the number of points did not exceed 30. Motivation evaluation parameters 2, 14, 5, 1, 19, 12, 8 at the beginning of the experiment were in the range from 30 to 50 points in both control and experimental groups. According to the results obtained after the experiment, we observe that the motivation of students for these parameters increased significantly in both groups. In the experimental group, the number of points increased by 29 on parameters 5 and 14 and by 35 points on parameter 1.

Analysis of the parameters of the manifestation of negative motivation shows that in the

control groups, the number of respondents who had positive changes in each parameter ranges from 10 points. As for the experimental group, the changes in points give fluctuations in the positive direction from 15 to 30 points.

In order to determine the internal students motivation level, the following rating is used: 0-5 points – low level of internal motivation; 6-14 points – the average level of internal motivation; 15-20 points – high level of internal motivation. The results of the survey are presented in figure 7.

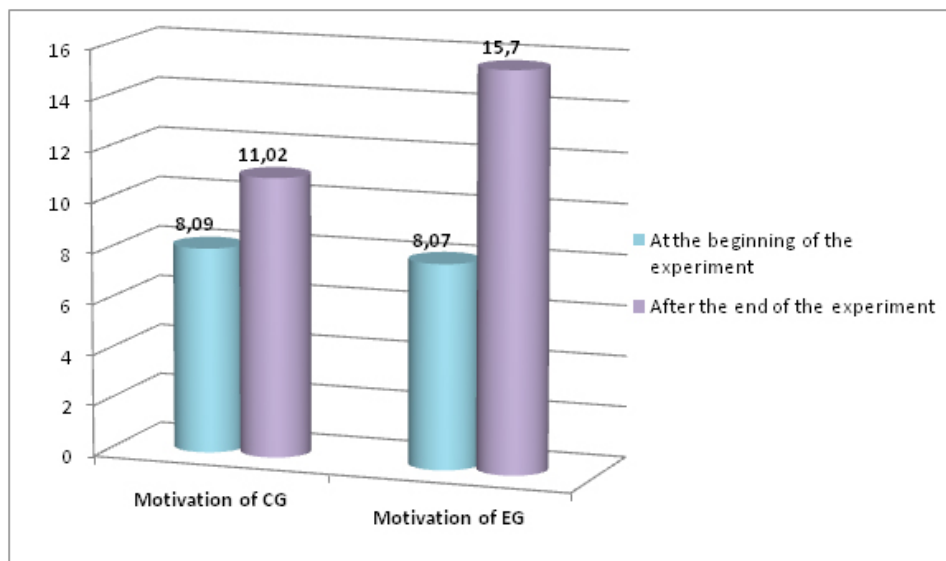


Figure 7. Comparative analysis of motivation levels among the students of CG and EG.

It is clear from the histogram (figure 7) that the motivation level among the students of both groups increased in CG, the GPA became 11.02, and in CG 15.7, but the implemented system of problems influenced more significantly the increase of motivation among the students of EG.

5. Discussion

According to the current programs [34] the topic Functions, their features and graphics is learned in high school which is specialized and students can learn this topic at the basic or specialized level. The actuality of developing cross-curricular problems while learning this topic at every level is determined first of all by the fact that such problems encourage the formation of mathematics competency as the key one [35]. In order to measure the level of students competency Pesakovic, Flogie, Abersek [35] offer to develop the appropriate tools. In our research competency-based mathematical problems that should be selected according to the learning specialization are the tools to determine students competency. For those students who learned Mathematics at the basic level of training, this subject is not specialized, thats why it is very important to motivate students to learn Mathematics by introducing cross-curricular problems that are connected with the students specialization (economic, natural, humanitarian).

For those students who learn Mathematics at the specialized level, it is important to learn how to use a mathematical modeling method that is realized through solving cross-curricular problems. It is necessary to learn mathematical modeling while learning every subject of the natural – mathematical cycle. This idea is proven in the works written by Little [36], Vlasenko et al. [37] who study the formation of connections between Mathematics and science and search for ways so that students of secondary schools do not consider Mathematics inappropriate questioning the use of its learning. The scientist believes that it is a STEM system that will help

to connect Mathematics with other school subjects. Rene de Cotret and Susanne Vincent [38] support this idea and consider the use of mathematics knowledge in everyday life, in particular, the integration of the subject knowledge in the project functions which is not in the area of the subject. Arpin supports the realization of cross-curricular connections which is beneficial both for the educational needs of a definite learning specialization and for Mathematics [39].

The analysis of the programs of learning subjects [34] where one can meet the problems connected with the topic Functions, their features, and graphics, encourages the creation of the classification to realize cross-curricular connections in time. Such approach of the authors is supported by the research done by Rivar [40] about setting different forms of relationship between the subjects: interdisciplinarity, multidisciplinary, transdisciplinarity, and interdisciplinarity in the strict meaning of this word. So, we suggest defining three types of connections in the research. The first of them is parallel learning. This connection takes place when the topic Functions, their features, and graphics in mathematics corresponds to the terms of learning these topics in such subjects as biology, physics, geography, chemistry, information technology that are connected with the functions. It is convenient to carry out students motivation in parallel while learning these subjects offering the same problems while learning the topic Functions, their features and graphics and while learning the topics indicated in table 1. The perspective connections arise during mathematics lessons while considering problems of cross-curricular character (table 2). Motivation improvement takes place because the importance of mathematical knowledge for such subjects as environment, economics, history of Ukraine, law, homeland defense, civil education, and art, which are specialized for students, is shown. At this stage, the use of cross-curricular problems encourages the creation of problematic situations. Such situations cause students' interest to learn Mathematics as a method to research the processes typical for the major (important) subject for the student in the future. The problems that were used during mathematics lessons as perspective connections can be suggested to students during the lessons on major subjects (history of Ukraine, law, homeland defense, civil education, environment, economics, art) according to the program, in the topics where these problems are actual (table 1). In this case, the third type of cross-curricular connections is realized – the use of mathematical modeling method while solving applied problems.

The authors of the research choose an applied problem as the main semantic item of the course. At the same time, we take a problem that is given outside Mathematics as an applied problem and it is solved by mathematical methods and ways. Applied problems can conditionally be divided into such ones where the mathematical problem is included in the problem condition (formalized) and those the solution of which does not imply building a mathematical model (non-formalized). The solution to the first ones is much easier than the solution to non-formalized problems and consequently consists of the following stages:

- analysis of the problem formation;
- search for the solution plan;
- plan fulfillment, checking, and research of the received solution;
- discussion (analysis) of the found method to solve problems aimed at finding out its rationality, ways to solve the problems using some other method or way.

While solving non-formalized problems the above-mentioned stages are supplemented due to the need to build mathematical models. That's why we refer to the stages of solving non-formalized applied problems the following ones:

- problem setting;
- translation of the problem conditions into the mathematics language;
- creation of the problem mathematical model;

- search for the plan to solve the problem inside the model;
- plan fulfillment, checking, and research of the received solution inside the model;
- interpretation of the received results;
- discussion (analysis) of the found method to solve problems aimed at finding out its rationality, ways to solve the problems using some other method or way.

Such problems enable learning new strategies of solving problems, developing analytical thinking among students that increases their motivation to learn mathematics.

6. Conclusions

Wide and multifaceted cross-curricular connections of Mathematics and other school subjects can become a guide for students to learn other subjects. Following the example of learning the topic Functions, their features and graphics it is demonstrated in the research the systematization of the competency-based problems focused on the increase of students motivation to learn mathematics.

The researchers recommend systematizing the problems of school subjects that are learned in a high specialized school and require the use of students knowledge in the topic Functions, their features, and graphics. Regarding the selection and systematization of cross-curricular problems, the researchers offer to select a system of competency-based problems for the learning subjects where the terms of the selected topic are used. Also, according to the recommendations given by the authors of the article, the formation of mathematical competency as the key one among the senior students takes place through the demonstration of cross-curricular connections of Mathematics and other learning subjects. It allows showing the advantages of mathematical modeling in the research of real processes.

It is recommended to classify the realization of cross-curricular connections in time in order to implement the offered system of problems in learning Mathematics. As a result of such classification, the authors of the article suggest setting three types of connections: parallel learning, perspective connections, use of the mathematical modeling method. Regarding the organization of cross-curricular connections aimed at forming motivation to learn Mathematics, it is suggested to allow the students to solve the selected system of problems independently in order to increase their understanding of using the topic that is learned and develop their skill to model different situations that in its turn increase their motivation to learn a topic. It is recommended to choose an applied problem as the main semantic item of the developed system. According to the recommendations given by the authors, applied problems can be conditionally divided into formalized, when a mathematical model is included in the problem condition and non-formalized that require building a mathematical model.

The positive results of implementing a system of cross-curricular problems in the process of learning Mathematics following the example of learning the topic Functions, their Features and Graphics are proven by the increase of students motivation level recorded during the experiment. Namely, in the experimental group, the positive motivation significantly increased, and the indicators of negative motives in teaching Mathematics decreased. Thus, the level of internal motivation of students increased by 7.63, which corresponds to a high level. However, as in the control group, there is an average level of intrinsic motivation of students.

In the future, the authors of this article plan focused on the development of competency-based teaching methods of such topics of the school course of Algebra as Derivative of a Function, Integral and its Application in a specialized school.

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