

## Model synthesis of nonlinear nonstationary dynamical systems in concentrating production using Volterra kernel transformation



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### Abstract

The features of the application of mathematical modeling of nonlinear dynamic systems such as “input-output”, which operate under the conditions of concentrating production, on the basis of finite sums of Volterra integral power series are considered.

Keywords: AUTOMATION, CONTROL SYSTEM, IDENTIFICATION, VOLTERRA KERNELS

The methods improvement of mathematical modeling and formation of automated control of nonlinear dynamical systems, which operate under the conditions of concentrating production, is one of the main approaches to the problem solving of increasing the efficiency of mining enterprises. The solutions to this problem in different aspects: the synthesis of mathematical models of processes, [1-4, 7], the formation of automated control [5, 6, 8, 9, 12], characteristics control of the processed ore raw materials [8, 9-11] and technological aggregates control optimization are offered in a significant number of works.

In modern conditions, on average of 5–8 ore technological varieties is processed on mining enterprises [9]. For the most effective concentration of each of these varieties with a maximum useful components extraction, and cost-effectiveness it is advisable to maintain the process certain operating parameters. However, the current system of mining operations

doesn't allow supplying the ore-dressing plant of the same type of ore for a long time. Therefore, during the operation, the aggregates technological parameters, as objects of control, will be constantly changing. Thus, for modeling and the control formation the nonstationarity of nonlinear dynamic objects of concentrating production should be considered. Due to the complexity of investigated processes, their nonlinearity, nonstationarity and large spatial extent, the conducting of large-scale experiments is difficult. As a result, the filling of missing information about the course of ore dressing technological processes is necessary to carry out by means of mathematical modeling. In the problem solving of models synthesis of nonlinear dynamic objects of concentrating production: grinding, classifying and separating; one of the most versatile approaches is the presentation of the system response to external influence in the form of a finite sum of Volterra integral power series [1-3,7]

$$\beta(t) = P_N[x(t)] \equiv \int_0^t \gamma_1(t, \zeta_1) x(\zeta_1) d\zeta_1 + \int_0^t \int_0^t \gamma_2(t, \zeta_1, \zeta_2) x(\zeta_1) d\zeta_1 d\zeta_2 + \int_0^t \dots \int_0^t \gamma_N(t, \zeta_1, \dots, \zeta_N) \prod_{i=1}^N x(\zeta_i) d\zeta_i, \quad t \in [0, T] \quad (1)$$

where  $\beta(t)$  – is the deviation of the iron content in the size fractions from steady-state value by supplying the input of the disturbance;  $x(t)$  – is the disturbance, depending on the mineralogical and technological characteristics of raw materials varieties;  $\gamma_N$  – Volterra kernels. In this case, the function  $\gamma_N$  – are transient characteristics of a dynamic system and are subject to identification. An important problem, which arises when using the finite sum of Volterra integral power series is the determination of the required  $N$  for the simulation of a specific nonlinear dynamic system. A common is the approach, assuming the use of the first 2-3 terms [1-3, 7].

In general, the problem of mathematical modeling of nonlinear dynamic systems based on finite sums of a Volterra series, according to recommendations [1],

is carried out in three stages. The first is to obtain a response of the dynamic system to the special input disturbances (combinations of Heaviside functions with deviating argument). Second, is to construct a composition of the dynamic system reactions, equal to  $n$ -th term of the Volterra series. The third implies the solution of integral equations of Volterra type I for the desired transient characteristics of the simulated process and its substitution into the original finite sum of Volterra series. The kernels recovery problem in the case of simulation of concentrating production nonstationary dynamical systems by finite sums of (1) with scalar input action is reduced to the solution of  $p$ -dimensional integral equation of Volterra type I [1-3]

$$Z_p \gamma_p = \sum_{i_1 + \dots + i_p} (-1)^{i_2 + \dots + i_{2(p/2)}} \frac{p!}{i_1! \dots i_p!} Z_{i_1 \dots i_p} \gamma_p = g_p \quad (2)$$

where

$$g_p = g_{i_1 \dots i_p}(t, s_1, \dots, s_p) = f_{i_1 \dots i_p}(t, \omega_1, \omega_1 + \omega_2, \dots, \omega_1 + \dots + \omega_p), \quad p \geq 1 \quad (3)$$

$$Z_{i_1 \dots i_p} \gamma_p = \int_0^{s_p} \dots \int_0^{s_p} \dots \int_{s_2}^{s_1} \dots \int_{s_2}^{s_1} \gamma_p(t, \zeta_1, \dots, \zeta_p) d\zeta_1 \dots d\zeta_p \quad (4)$$

In case of vector input disturbance, it is suggested to use the finite sums of Volterra type in [1]

$$\beta(t) = \sum_{p=1}^q \sum_{1 \leq i_1 \leq \dots \leq i_p \leq m} f_{i_1, \dots, i_p}(t) \quad (5)$$

where

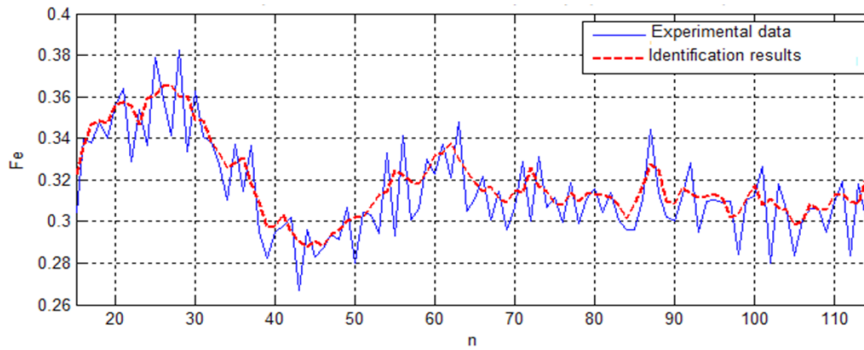
$$f_{i_1, \dots, i_p}(t) = \int_0^t \dots \int_0^t \gamma_{i_1, \dots, i_p}(t, \zeta_1, \dots, \zeta_p) x_{i_1}(\zeta_1) \dots x_{i_p}(\zeta_p) d\zeta_1 \dots d\zeta_p, t \in [0, T] \quad (6)$$

The problem solution of Volterra kernels identification is produced using mathematical modeling, which proposed in [1-3,7]. The efficiency verification of Volterra kernels identification algorithms is carried out by computer simulation on the basis of passive experiment.

It should be noted that the ore beneficiation control processes are characterized by significant spatial distribution and a significant number of parameters [4,6,12].

The dynamics can first be decomposed into a series of kernels whereby previous spatial and temporal kernels separation is performed as shown in Fig. 2 [4].

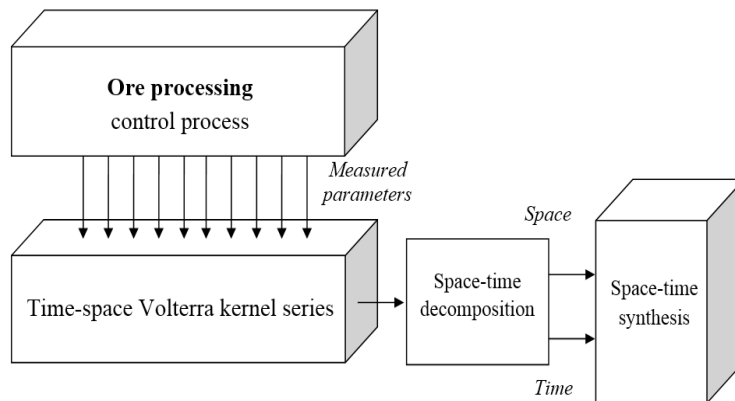
$$g_r(\cdot) = \sum_{i=1}^n \sum_{j_1=1}^m \dots \sum_{j_r=1}^m \sum_{k_1=1}^l \dots \sum_{k_r=1}^l \theta_{i j_1 \dots j_r k_1 \dots k_r}^{(r)} \times \prod_{s=1}^r \phi_i(x) \varphi_{j_s}(\xi_s) \psi_{k_s}(\tau_s), r = 1, \dots, R, \quad (7)$$



**Figure 1.** The result of ore beneficiation process identification

After the estimation of unknown parameters the kernels can be transformed using the spatio-temporal synthesis. The quality of the simulation will be improved by increasing the number of kernels. Using a plurality of kernels, allows to approximate a wide range of non-linear systems of concentrating produc-

tion with distributed parameters. Such approach allows to increase the simulation accuracy of the dynamic effect of input actions in the concentrated points of technological process to the output characteristics of processed ore material, which distributed over the technological line.



**Figure 2.** Simulation of the ore beneficiation process based on the representation using Volterra kernels

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