Augmented reality as a tool for visualization of ultrasound propagation in heterogeneous media based on the *k*-space method

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Abstract. For programming the AR tools, interactive objects and creating the markers, the method of fiber spaces (*k*-space) for modeling of ultrasonic wave propagation in an inhomogeneous medium using coarse grids, with maintaining the required accuracy was used. The algorithm and tools of augmented reality were introduced into the adaptive control system of the pulp gas phase in the iron ore flotation process using a control action on the basis of high-energy ultrasound dynamic effects generated by ultrasonic phased arrays. The tools of augmented reality based on *k*-space methods allow to facilitate wider adoption of ultrasound technology and visualize the ultra-sound propagation in heterogeneous media by providing a specific correspondence between the ultrasound data acquired in real-time and a sufficiently detailed augmented 3D scene. The tools of augmented reality allow seeing the field of ultrasound propagation, its characteristics, as well as the effect of the dynamic effects of ultrasound on the change in the gas phase during the flotation process.

Keywords: Augmented reality, ultrasound propagation, *k*-space method.

1 Introduction

Nowadays, the growth of applications of augmented reality (AR) can be attributed to solutions, which allow to visualize products and their characteristics, add some interactive objects which allow looking inside the processes, etc.

Every year applications using augmented reality are gaining more and more popularity. It is used in various fields of activity: production, repair [20], training [21], sales [7], marketing, exhibitions, user guides [8], remote maintenance [17], and navigation [4]. To build a functioning system, a sufficiently powerful platform is needed, which can be a modern mobile device [9], due to their widespread and ever-growing capabilities. To draw virtual objects, markers are used that are located in the surrounding space. They are located and analyzed by special software [2; 6; 16].

For the control of the basic technological parameters and mineral beneficiation process control, an important task is to control the parameters of complex heterogeneous mediums, including solid, liquid and gas phases.

In [10; 11; 12; 13; 14; 15] automatic control system of gas bubble size distribution based on the ultrasonic phased array technology, which allows to implement the efficient control of pulp gas phase composition, adjust the aeration degree, increase the flotation speed, increase the concentrate quality and energy efficiency of the entire mineral processing process is proposed. In this research, the ultrasound as the main tool was used.

2 Materials and methods

To facilitate wider adoption of ultrasound technology we use the tools of augmented-reality to visualize the ultrasound propagation in heterogeneous media by providing a specific correspondence between the ultrasound data acquired in real-time and a sufficiently detailed augmented 3D scene. We have established a tablet-based system for visualizing the propagation of high-energy ultrasound in a heterogeneous medium in the process of froth flotation using augmented-reality techniques in conjunction with the streaming data about ultrasound characteristics provided by phased array based on *k*-space method. This system gives the operator visual feedback as to the location of the ultrasonic spot generated by the elements of the phased array, the characteristics of the ultrasound beam, and look inside the flotation tank.

For programming, the AR tools, interactive objects and creating the markers, the method of fiber spaces (*k*-space) for modeling of ultrasonic wave propagation in an inhomogeneous medium using coarse grids, with maintaining the required accuracy was used [1; 11; 19].

We describe the ultrasonic waves propagation depending on the mass conservation equations, momentum conservation law and the equation of state using the first order dual equations, which can be summarized as follows [1; 5]

$$\frac{\partial p(\bar{x},t)}{\partial t} + \rho(\bar{x}) c^2(\bar{x}) \nabla v(\bar{x},t) = -\alpha(\bar{x}) p(\bar{x},t), \tag{1}$$

$$\rho(\bar{x}) \frac{\partial v(\bar{x},t)}{\partial t} + \nabla p(\bar{x},t) = 0, \tag{2}$$

where $p(\bar{x},t)$ – the time and space dependent ultrasound pressure perturbations (*x* – 3D Cartesian axis (*x, y, z*)); $\rho(\bar{x})$ is the spatially dependent density; $c(\bar{x})$ is the spatial dependent sound speed; $v(\bar{x},t)$ is the velocity of the particle and $\alpha(\bar{x})$ is the absorption coefficient which equivalent to the inverse of the relaxation time.

Let’s represent all absorption effects with one relaxation time. From (2), the simplified equation can be written as follows

$$\frac{\partial v(\bar{x},t)}{\partial t} = \frac{-\nabla p(\bar{x},t)}{\rho(\bar{x})} \tag{3}$$

We differentiate (1) with respect to time and variations in (2), and the final equation can be represented as follows

$$\frac{\partial^2 p(\bar{x},t)}{\partial t^2}+\rho(\bar{x})c^2(\bar{x})\frac{\partial}{\partial t}\nabla\partial v(\bar{x},t)=-\alpha(\bar{x})\frac{\partial p(\bar{x},t)}{\partial t}, \tag{4}$$

$$p(\bar{x},t)\frac{\partial}{\partial t}\nabla v(\bar{x},t)+\frac{\partial v(\bar{x},t)}{\partial t}\nabla\rho(\bar{x})+\nabla^2 p(\bar{x},t)=0, \tag{5}$$

Taking into account the permutations (4)

$$\frac{\partial}{\partial t}\nabla v(\bar{x},t)=-\left(\frac{\alpha(\bar{x})}{\rho(\bar{x})c^2(\bar{x})}\frac{\partial p(\bar{x},t)}{\partial t}+\frac{1}{\rho(\bar{x})c^2(\bar{x})}+\frac{\partial^2 p(\bar{x},t)}{\partial t^2}\right) \tag{6}$$

By substituting this equation in (5), we obtain

$$\frac{-\alpha(\bar{x})}{c^2(\bar{x})}\frac{\partial p(\bar{x},t)}{\partial t}-\frac{\partial^2 p(\bar{x},t)}{c^2(\bar{x})\partial t^2}-\frac{1}{\rho(\bar{x})}\nabla p(\bar{x},t)\nabla p(\bar{x})+\nabla^2 p(\bar{x},t)=0, \tag{7}$$

The simplification of the pressure deviation to the density gradient can be represented as follows

$$\nabla\left(\frac{\nabla p(\bar{x},t)}{\rho(\bar{x})}\right)=\frac{\nabla^2 p(\bar{x},t)}{\rho(\bar{x})}-\frac{\nabla p(\bar{x},t)\nabla p(\bar{x})}{\rho(\bar{x})^2}, \tag{8}$$

Taking into account (7), eq. (8) can be represented as follows

$$\nabla\left(\frac{1}{\rho(\bar{x})}\nabla p(\bar{x},t)\right)-\frac{1}{\rho(\bar{x})c^2(\bar{x})}\frac{\partial^2 p(\bar{x},t)}{\partial t^2}=\frac{\alpha(\bar{x})}{\rho(\bar{x})c^2(\bar{x})}\frac{\partial p(\bar{x},t)}{\partial t}, \tag{9}$$

This is a linear wave equation of ultrasonic wave propagation in the heterogeneous medium with the absorption parameters.

Let's simplify (9) by separating the parameters of the sound velocity $c(\bar{x})$ and density $\rho(\bar{x})$ from the second derivatives of pressure taking into account the spatial and temporal variables to solve the problem of ultrasound propagation using the fiber space method.

The original equation can be written in the form

$$\nabla\left(\frac{1}{\rho(\bar{x})}\nabla p(\bar{x},t)\right)-\frac{1}{\rho(\bar{x})c^2(\bar{x})}\frac{\partial^2 p(\bar{x},t)}{\partial t^2}=0, \tag{10}$$

The normalized pressure can be represented as follows

$$\psi(\bar{x},t)=\frac{p(\bar{x},t)}{\sqrt{p(\bar{x})}}$$

By substituting this equation in (10) we obtain

$$\nabla\left(\frac{1}{\rho(\bar{x})}\nabla p^{1/2}(\bar{x},t)\psi(\bar{x},t)\right)=\frac{\rho^{1/2}(\bar{x})}{\rho(\bar{x})c^2(\bar{x})}\frac{\partial^2\psi(\bar{x},t)}{\partial t^2}$$

After simplifying

$$\nabla^2\psi(\bar{x},t)-\rho^{1/2}(\bar{x})\psi(\bar{x},t)\nabla^2\rho^{1/2}(\bar{x})=\frac{1}{c^2(\bar{x})}\frac{\partial^2\psi(\bar{x},t)}{\partial t^2}$$

Taking into account further simplifications the equation takes the form

$$\nabla^2\psi(\bar{x},t)-\frac{1}{c_0^2}\frac{\partial^2\psi(\bar{x},t)}{\partial t^2}=\frac{1}{c_0^2}\left[c_0^2\rho^{1/2}(\bar{x})\left(\nabla^2\rho^{1/2}(\bar{x})\right)\psi(\bar{x},t)+\left(\frac{c_0^2}{c^2(\bar{x})}-1\right)\frac{\partial^2\psi(\bar{x},t)}{\partial t^2}\right]$$

Even more, simplification can be obtained by determining the functions $q(r,\ t)$ and $v(r,\ t)$ efficient sources, which can be summarized as follows

$$q(\bar{x},t)=c_0^2\rho^{1/2}(\bar{x})\psi(\bar{x},t)\nabla^2\rho^{-1/2}(\bar{x})$$

$$v(\bar{x},t)=\left(\frac{c_0^2}{c^2(\bar{x},t)}-1\right)\psi(\bar{x},t)$$

By simplifying (11) we obtain

$$\nabla^2\psi(\bar{x},t)-\frac{1}{c_0^2}\frac{\partial^2\psi(\bar{x},t)}{\partial t^2}=\frac{1}{c_0^2}\left(q(\bar{x},t)+\frac{\partial^2 v(\bar{x},t)}{\partial t^2}\right), \tag{11}$$

This equation can be easily transformed into the frequency domain by using the three-dimensional spatial Fourier transform as follows

$$k^2F(\mathrm{k,t})-\frac{1}{c_0^2}\frac{\partial^2F(\mathrm{k,t})}{\partial \mathrm{t}^2}=\frac{1}{c_0^2}\left(Q(\mathrm{k,t})+\frac{\partial^2V(\mathrm{k,t})}{\partial \mathrm{t}^2}\right), \tag{12}$$

where $F(k,\ t)$, $Q(k,\ t)$ and $V(k,\ t)$ – three-dimensional spatial Fourier transformation of values $\psi(\bar{x},t)$, $q(\bar{x},t)$ and $v(\bar{x},t)$ respectively. Equation (12) satisfies the total wavefield and is defined as the sum of the incident and scattered field $\psi(\bar{x},t)=\psi_i(\bar{x},t)+\psi_s(\bar{x},t)$, and the scattered wave field.

$$\nabla^2\psi(\bar{x},t)-\frac{1}{c_0^2}\frac{\partial^2\psi(\bar{x},t)}{\partial t^2}=0\ .$$

For the case of an inhomogeneous medium, we introduce an additional source $w(\bar{x},t)=\psi_s(\bar{x},t)+v(\bar{x},t)$ and by substituting it into (13) we obtain the following expression

$$\frac{\partial^2 W(k,t)}{\partial t^2}=k^2c_0^2\left[W(k,t)-V(k,t)\right]-Q(k,t), \tag{13}$$

where $V(k,t)=F\left[\left(1-\frac{c^2(\bar{x})}{c_0^2}\right)\left(\psi_i(\bar{x},t)+w(\bar{x},t)\right)\right]$;

$$Q(k,t)=c_0^2F\left[\sqrt{\rho(\bar{x})}\nabla^2\rho^{1/2}(\bar{x})=\left[\psi_i(\bar{x},t)+w(\bar{x},t)-v(\bar{x},t)\right]\right]$$

where F is a spatial Fourier transform.

Let's use the substandard finite difference approach to solve this equation [10; 15]. Discretization of the time derivative gives

$$\begin{gathered}W(k,t+\Delta t)-2W(k,t)+W(k,t-\Delta t)=4\sin^2\left(\frac{c_0k\Delta t}{2}\right)\times\\ \times\left[V(k,t)-W(k,t)-\frac{Q(k,t)}{c_0^2k^2}\right]\end{gathered}, \tag{14}$$

Consider the wave equation on the grayscale for the fiber space method (*k*-space), which includes the non-linear characteristic of ultrasound, which can be represented as follows [10]:

$$\nabla^2\psi(\bar{x},t)-\sqrt{\rho(\bar{x})}\psi(\bar{x},t)\nabla^2\frac{1}{\sqrt{\rho(\bar{x})}}-\frac{1}{c^2(\bar{x})}\frac{\partial^2\psi(\bar{x},t)}{\partial t^2}=-\frac{\beta(\bar{x})}{\sqrt{\rho_0}c_0^4}\frac{\partial^2\psi^2(\bar{x},t)}{\partial t^2}$$

where $\psi^2(\bar{x},t)$ is the nonlinearity source, $\beta(\bar{x})$ is the nonlinearity coefficient. The harmonic oscillations equation can be represented as follows

$$\frac{\partial^2 W2(\bar{k},t)}{\partial t^2}=\left(c_0^2k^2\right)\left(VNL2(\bar{k},t)-W2(\bar{k},t)\right)-Q(\bar{k},t), \tag{15}$$

where $w2(\bar{x},t)=\psi_s(\bar{x},t)+v_{NL2}(\bar{x},t)$ – additional source; $W2(\bar{k},t)$ is a spatial Fourier transform.

$$v_{NL2}(\bar{x},t)=\left(\frac{c_0^2}{c^2(\bar{x})}-1\right)\psi(\bar{x},t)-\frac{\beta(\bar{x})}{\sqrt{\rho_0}c_0^2}\left(\psi_s^2(\bar{x},t)+2\psi_s(\bar{x},t)\psi_i(\bar{x},t)\right)$$

After the spatial Fourier transformation, the equation can be expressed as follows

$$VNL2(\bar{k},t)=\mathrm{F}\left\{\begin{array}{l}\left(\frac{c_0^2}{c^2(\bar{x})}-1\right)\left[\psi_i(\bar{x},t)+w2(\bar{x},t)-\frac{\beta(\bar{x})}{\sqrt{\rho_0}c_0^2}\left(\psi_s^2(\bar{x},t)-2\psi_s(\bar{x},t)\psi_i(\bar{x},t)\right)\right.\\-\frac{\beta(\bar{x})}{\sqrt{\rho_0}c_0^2}\left(\psi_s^2(\bar{x},t)-2\psi_s(\bar{x},t)\psi_i(\bar{x},t)\right)\end{array}\right\};$$

$$Q(k,t)=\mathrm{F}\left[c_0^2\sqrt{\rho(\bar{x})}\nabla^2\left(\frac{1}{\sqrt{\rho(\bar{x})}}\right)\left[\psi_i(\bar{x},t)-w2(\bar{x},t)-v_{NL2}(\bar{x},t)\right]\right];$$

The introduction of the nonlinearity term in fiber space method makes it easier to calculate the actual relief temperature in heterogeneous large scale models.

Let's simulate the ultrasonic pressure field propagation in a heterogeneous medium using k-Wave toolbox (Matlab) which is designed for time domain ultrasound simulations in complex media like heterogeneous pulp. The simulation functions of this software are based on the k-space method and are both fast and easy to use [3; 18].

The net pressure of all piezoelectric elements can be obtained by adding the effects of each source and written in the form

$$P_{net}(x,y,z)=\sum_{i=1}^{n}p_i(x,y,z)\,. \tag{16}$$

Due to attenuation, the useful power at the point (x, y, z) is given by [18]

$$q(x,y,z)=\frac{\alpha P_{net}^2(x,y,z)}{\rho c}\,, \tag{17}$$

The total energy at a point (x, y, z) is given by

$$I(x,y,z)=\frac{p^2(x,y,z)}{2\rho c}\,, \tag{18}$$

where $I(x, y, z)$ – intensity at the point (x, y, z), W/m^2.

The results of the ultrasonic wave propagation through a heterogeneous medium with density ρ = 1500 g/m^3, for source strength of 1 MPa and tone burst frequency of 1 MHz for 16-element, phased array with a focus distance of 20 mm are shown on Fig. 1. The central slice absorption distribution in grayscale as a background and the square of the pressure distribution on the surface of this background are shown.

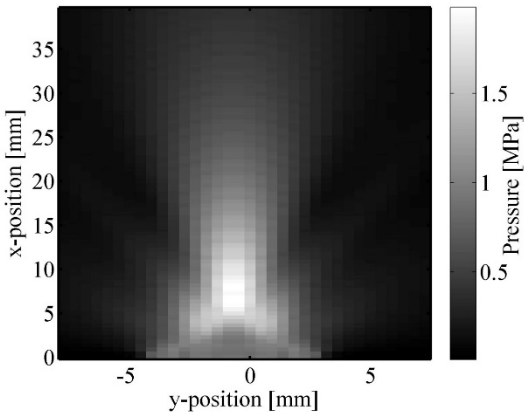

Fig. 1. Total beam pattern using the maximum of recorded pressure

The final pressure field (a), the maximum pressure (b) and standard pressure (c) of the beam are shown in Fig. 2. The transducer focus and sidelobes are visible.

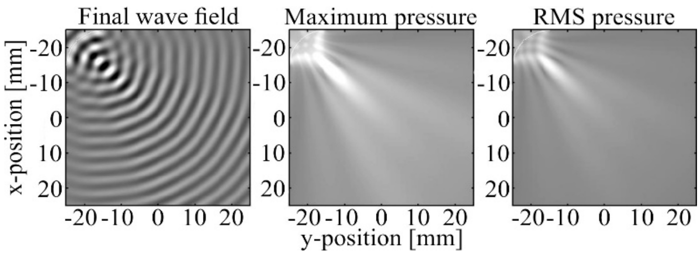

Fig. 2. Ultrasonic wave propagation in a heterogeneous medium: a) the final pressure field, b) the maximum pressure c) the RMS pressure

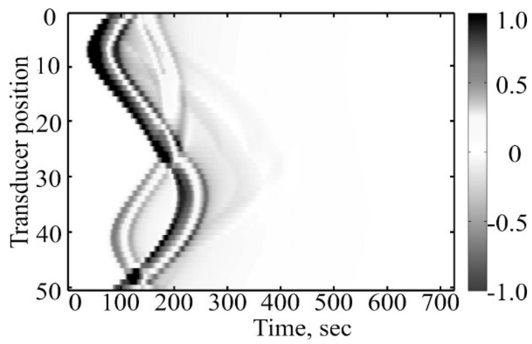

Fig. 3. The shape of the main wavefront

The linear cross-section of the focus in the x direction is shown in Fig. 4: 1) for the single source; 2) simulation by a k-space method in the water; 3) in a heterogeneous medium.

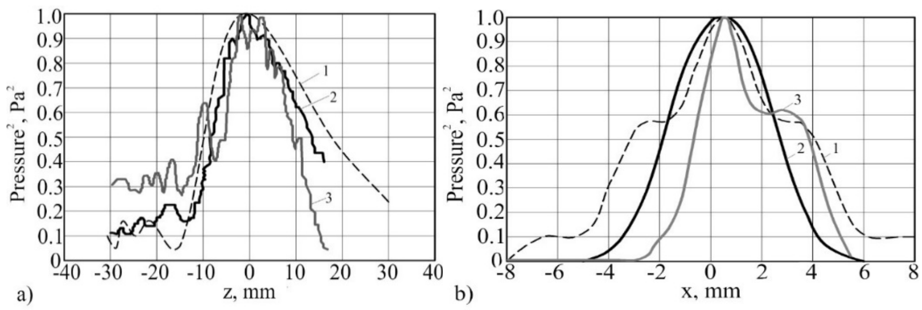

Fig. 4. The simulation results comparison of the normalized square of pressure for: 1) a simple screened source, 2) modeling by the *k*-space method in a homogeneous medium (water) and 3) in the inhomogeneous medium (pulp) along the axis: a) – *z* and b) – *x*.

3 Results

Based on the obtained results, augmented reality tools (interactive objects, markers) (see Figure 5, 6) were developed in the AR Editor, which are implemented based on the following algorithm:

- STEP 1. Image Capture
  - STEP 1.1 Connecting the camera
  - STEP 1.2 Capturing video from the camera
  - STEP 1.3 Reading video frame frames
- STEP 2. Recognition of special points and descriptors
  - STEP 2.1 Finding special points on the image
  - STEP 2.2 Calculation ─ singular points descriptors
- STEP 3. Comparison of descriptors calculated in STEP 2.2 with the database of marker descriptors. Getting id markers, point vector
- STEP 4. Reproduction of relevant content
  - STEP 4.1. If the returned number of points in the vector is greater than the specified value, then go to STEP 4.2. Otherwise, in STEP 4.4.
  - STEP 4.2 Getting the content of the marker from the database
  - STEP 4.3 Display the contents of the marker on the image
  - STEP 4.4 Displaying the image on the screen

The developed algorithm and tools of augmented reality were introduced into the adaptive control system of the pulp gas phase in the iron ore flotation process using a control action on the basis of high-energy ultrasound dynamic effects generated by ultrasonic phased arrays (see Figure 7). The adaptive control system based on the ultrasonic phased array is placed on the wall of the flotation machine. In accordance with the above method and the developed algorithm, the system generates a marker in which the current state of the system is embedded. The tools of augmented reality allow seeing the field of ultrasound propagation, its characteristics, as well as the effect of the dynamic effects of ultrasound on the change in the gas phase during the flotation process.

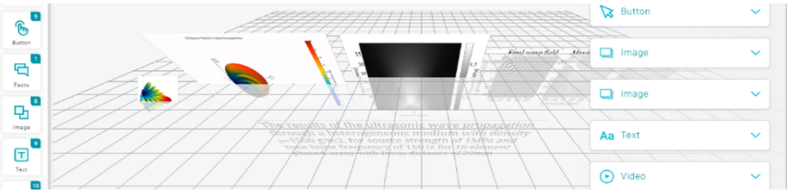

Fig. 5. AR tools (interactive objects, markers) developing in the AR Editor

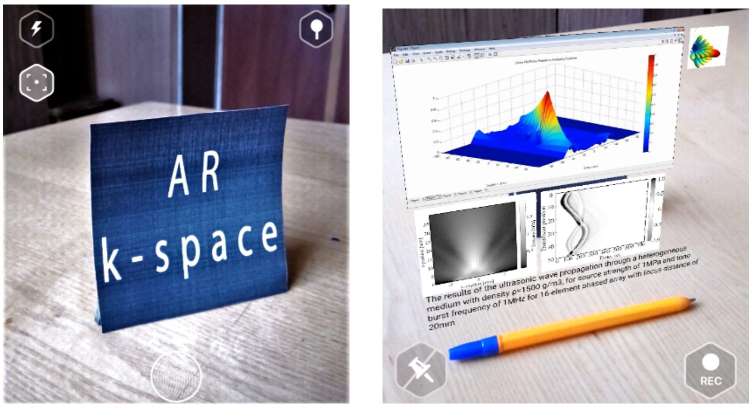

Fig. 6. AR k-space marker and displaying the contents of the marker on the image

Fig. 7. Augmented reality tools in the adaptive control system of the pulp gas phase of the iron ore flotation process

Conclusions

To build a model of the ultrasonic field in a randomly inhomogeneous medium, the fiber spaces method (*k*-space), which increased the accuracy of parameter estimation field is used. The tools of augmented reality based on *k*-space methods were developed, which allow to facilitate wider adoption of ultrasound technology and visualize the ultrasound propagation in heterogeneous media by providing a specific correspondence between the ultrasound data acquired in real-time and a sufficiently detailed augmented 3D scene.

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